Help and Factionalism in Politics and Organizations *

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Abstract Whether in electoral politics or promotions within organizations, players often face the dilemma of whether to enter the contest or to assist other candidates. This paper analyzes incentives in a rank-order tournament when the winner, apart from earning the "first prize," also has control over a "second prize" that he can distribute to his supporters. Some players may then be encouraged to help others in exchange for paybacks, resulting in factionalism, with leaders, solo contestants and supporters of other candidates sorted by ability. The number and the size of factions depend on the structure of the contest, which can be manipulated to provide optimal incentives for effort coordination as required by political objectives or production technology.

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I. Introduction

In politics, whether in a democracy or otherwise, there is often no shortage of aspirants for public offices. In the United States, presidential hopefuls abound, but only a relatively small number of candidates actually enter the race every four years. Some simply stay out of the contest or drop out during the process, while others choose to support the campaigns of other contenders. In less open political systems, when to stand up and be counted and when to lie low and play loyal crony can be an even more tricky decision, sometimes with more than a career at stake. So, when there are players of different calibres in this political game, who would step into the ring and who would resign themselves to a supporting role? Given that the strength of a campaign depends both on the ability of the candidate and the support he receives, what determines the frontrunners and the underdogs? These are interesting questions not just in the public arena but also within organizations as well. Even though candidacies for promotion to senior positions are rarely openly announced and battle lines are usually not clearly drawn, the jockeying for position and lobbying for support are often played out in the corporate boardroom or even academic institutions with as much ferocity and intrigue as in electoral politics, as individuals sort themselves into different roles within and across contending factions. How then can we understand the dynamics and the outcome of this process?

Whether it is a contest for public office or promotion within an organization, the payoffs are usually fixed and entitlement to these rewards generally depends on relative performance only (at the ballot box or by other assessment mechanisms). As such, the rank-order tournament appears to offer an appropriate theoretical framework for the analysis of incentives within the game. But while the economic efficiency of the tournament in eliciting effort has been well discussed in the literature (e.g. Lazear and Rosen 1981, Holmstrom 1982, Green and Stokey 1983, Nalebuff and Stiglitz 1983, Mookherjee 1984, and Malcomson 1986), the conventional model does not offer a ready explanation for helping or cooperative behavior. Indeed, as Lazear (1989) points out, because hampering an opponent has a similar effect on the outcome as enhancing a candidate's own performance, competitive pressure in a contest can result in negative behavior among candidates, ranging from non-cooperation to outright sabotage of other contestants' work, which obviously compromises efficiency. Chen (2003) and Munster (2004) extend the Lazear sabotage model to settings with multiple heterogeneous players and find that because of the tendency for players to gang up on the frontrunner, more productive candidates may not enjoy a higher chance of winning. This can further contribute to inefficiency if sorting by ability into different positions is important in production.

Because of the inevitably antagonistic nature of a contest, strategic interaction among agents tends to have negative results. Eliciting cooperation among agents appears to be difficult if not impossible. Although Lazear (1989) raises the possibility of "sainthood" (in which a player derives utility from helping others at the expense of his own chance of winning), his point is that even saints may fail when tempted with the opportunity to sabotage the competition. Similarly, Grund and Sliwha (2002) find that not even "compassion," an aversion for pay inequity felt by winners of contests, can result in cooperative behavior in a contest. When teamwork in the workplace is discussed in the literature, it is usually in a context that does not involve compensation by relative performance. For example, Itoh (1993) shows that when agents can more effectively monitor one-another's behavior than the principal can, the optimum solution to the agency problem may involve the principal designing a coalition-proof contract to induce Pareto efficient efforts while delegating the implementation and enforcement to the agents themselves (through side-contracting). In another paper, Itoh (1991) also finds that help for coworkers can be induced by conditioning a worker's pay not only on the output of his own task but also on those of his coworkers. In neither case does the conventional tournament fit the requirements for an effective scheme in promoting teamwork. More explicitly, Che and Yoo (2001) compares relative performance evaluation (RPE) and joint performance evaluation (JPE) in a dynamic setting where implicit incentives are generated by repeated interaction of the agents. They conclude that compensation schemes based on JPE dominate those based on RPE because it is better able to encourage coordination among workers in a long-term relationship.

It would likely be futile to search for sainthood or compassion in politics or in the boardroom, much less appeal to them for an explanation for the behavior of politicians or corporate ladder-climbers. Yet, incentives for the allocation of efforts can look very different once we recognise that in a real-world contest, the winner often has at his disposal certain resources that can be distributed as rewards for friends and supporters. Winners of political elections are often entitled to make political appointments or hand out patronage jobs as paybacks for contributions of various sorts to their campaigns, while within a firm a newly appointed CEO usually has the authority to pick members of his senior management team, and more often than not, they would be his "close associates." Given these payoffs, each potential candidate in a contest must now decide whether to focus on improving his own chance at winning the top position, or devote part or all of his efforts to help another candidate. Such a decision obviously depends on the relative magnitudes of the payoffs (of winning the "first prize" vs. getting a share of the "second prize," the latter also affected by the number of "helpers" who must be rewarded). It would also depend on the worker's ability relative to that of other candidates, as well as on the way in which help from supporters is translated into better performance for the candidate. Under plausible assumptions and given a spread of the abilities of the players, we can show that a subgame perfect equilibrium always exists in which faction leaders, unassisted contestants and helpers are sorted by ability. In addition, by changing the rewards of the contest, different equilibrium structures of factions can be induced, ranging from a grand coalition of all players assisting a single candidate to every player pursuing his own chance at the top. These results can potentially offer insights into how tournament schemes can be designed to elicit optimal effort coordination, or how corporate cultures can arise from differences in production technologies.

The choice of the term “factionalism” over the more commonly encountered “coalition formation” in the title of the paper is deliberate. The latter concept is often found in the game theoretic literature that focuses on how coalition formation can affect final resource allocation. One line of this research investigates the division of surpluses and the efficiency of blocking coalitions in multi-person bargaining games (e.g. Chatterjee et al. 1993, Bloch 1996, Okada 1996, and Seidmann and Winter 1998). Another approach explores coalition formation as a way of implementing the core of a coalition game in a noncooperative manner (e.g. Perry and Reny 1994). This paper, on the other hand, tries to characterize the coalitions that would be observed when all agents move unilaterally, abstracting from bargaining and redistribution among agents. Moreover, this paper attempts the problem within the structured setting of a rank-order tournament. Therefore, it does not presume to offer a general solution to the coalition formation problem, and is only tangentially related to the game theoretic literature on the topic.

II. The Model

A. Assumptions

Consider an organization with N heterogeneous workers, each of whom is a potential candidate in a rank-order tournament. Each worker i is endowed with ability $t_i > 0$, and one unit of indivisible effort which he can use either to raise his own performance or to help another worker.[1] The effect of "help" and the stochastic structure of the contest can be summarized by the following winning probability of player i:

$$P(i\,wins) = \frac{t_i f\left(\sum_{h=1}^{N} e_{hi}\right)}{\sum_{k=1}^{N} t_k f\left(\sum_{h=1}^{N} e_{hk}\right)}, \tag{1}$$

where e_{hi} = the amount of help received by player i from player h, for $i,h = 1,\ldots,N$, and $f(.)$ is a monotonic increasing function. This specification of the winning probability is similar to those in Chen (2003) and Rosen (1986). The "multiplier" effect of the ability of the candidate receiving help can be justified by Rosen (1983), which suggests that the productivity of a senior worker is transmitted down the chain of command to those within his span of control. It is also consistent with the empirical observation in Hamilton et al. (2003) that the most productive member in a team has a greater influence on the team's productivity.

The contest is played out in two stages. In the first stage, players decide whether to present themselves as candidates in the contest and invest their effort on their own chances, or to help another candidate win the contest. In the second stage, once the candidates have identified themselves, helpers will decide on which candidate to help, and a candidate will accept any offer of help. Each helper makes the decision on his own, taking the decisions of

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[1] By assuming effort to be fixed, we are abstracting from the work incentive effect of a rank-order tournament and focus instead on the allocation of help within an organization.

all others as given. A faction arises when one or more players contribute their efforts in helping the same candidate who becomes the leader of that faction.[2]

Because each player's endowed effort is indivisible, there is no overlap in faction membership. As a result, the sum of helping efforts in equation (1) simply gives the size of the faction headed by player i. With the helpers out of contention for the top prize, let M ($\leq N$) represents the number of candidates. If we further simplify by assuming $f(x) = x$, then equation (1) reduces to

$$P(i\ wins) = \frac{t_i n_i}{\sum_{k=1}^{m} t_k n_k}, \tag{2}$$

where n_i = number of members in faction i (including player i, the faction leader).

The first-stage decision on whether to become a candidate or a helper depends on each player's anticipation of the distribution of helpers in the second stage and, of course, on the payoffs of the contest. In the most basic contest with multiple agents, there may be two or more fixed prizes to be awarded according to the rank order of the performance of the candidates.[3] We depart from the conventional setup in that, in additional to the winner's prize (W_1) and the loser's payoffs (normalized to zero), there is an additional fixed "second prize" ($W_2 < W_1$) that is distributed at the discretion of, but cannot be pocketed by, the contest winner. We assume that this amount would be divided equally among those who have helped

[2] Formation of a faction can be either implicit or explicit. It is interesting to note that workers in Microsoft can choose the project they want to join, and workers in GE can choose which division to work in. While there can be many reasons for such policies, they certainly tend to encourage camaraderie, cooperation, and perhaps factionalism as well.

[3] There can be potentially as many prizes as there are players, as in Green and Stokey (1983), but as Nalebuff and Stiglitz (1983) shows, three different prizes often suffice in an optimal scheme.

the winner in the contest. If the winner received no help from anyone (other than himself), then there is no reason for him to show favoritism towards any individual, and the "second prize" will be distributed equally among all other workers.[4]

To abstract from bargaining that can greatly complicate the problem, we preclude any side-payments that redistribute the prizes among members within and across factions. In many contexts, this may in fact be the reasonable assumption. A newly elevated CEO promoting his lieutenants to senior positions, or an elected politician repaying political debts with appointments in a new administration, usually cannot legally involve kickbacks or bribes paid in either direction. While illicit or informal transfers cannot be ruled out, we do not believe we are sacrificing too much generality with this simplifying assumption.

B. The Equilibrium

The two-stage problem constitutes an almost perfect information sequential game for all players. We shall look for the (pure strategy) subgame perfect equilibrium using backward induction, solving first for the equilibrium allocation of helpers before considering the sorting of players into leading and supporting roles.

1. Allocation of Helpers Across Factions

Consider the second-stage problem when M ($\leq N$) players have declared their candidacies for the first prize. If $M = 0$, then there are no contestants, and everyone gets zero. If $M = N$, then there are no helpers to allocate and the expected payoff for each player will be given by equation (2) for $n_i = 1$ for all $i = 1, \ldots, N$. If $0 < M < N$, the remaining $N - M$ players are helpers and will seek out and join the faction that maximizes their expected payoff, given

[4] If the total number of players is large, then the expected payoff that other players can get in case a solo contestant wins the contest is independent of marginal reallocation of helpers across factions. It becomes a constant in each player's (other than the solo contestant's) objective and will not affect his maximizing solution.

the choices of all other helpers. Free mobility across factions is assumed but randomization is not allowed. An equilibrium helper allocation is a list of helping effort $n = (n_1, \dots, n_M)$ such that:

$$n_i \geq 1 \tag{3}$$

for all $i = 1, \dots, M$;

$$\frac{t_i n_i}{\sum_{k=1}^{M} t_k n_k} \frac{W_2}{n_i - 1} \geq \frac{t_j \left(n_j + 1\right)}{\sum_{k=1}^{M} t_k n_k - t_i + t_j} \frac{W_2}{n_j} \tag{4}$$

for all j, and for all i such that $n_i > 1$; and

$$\sum_{k=1}^{M} n_k = N \,. \tag{5}$$

Condition (3) states that each candidate must receive at least one unit of helping effort, namely, his own. Condition (4) maintains that, for any faction with at least one helper, no helper would find it optimal to unilaterally deviate and join another faction. Equation (5) is a an add-up constraint requiring that efforts across factions sum up to total effort available. Condition (4) can be rearranged to give

$$\frac{\sum_{k=1}^{M} t_k n_k - t_i + t_j}{\sum_{k=1}^{M} t_k n_k} \frac{t_i n_i}{n_i - 1} \geq \frac{t_j \left(n_j + 1\right)}{n_j} .$$

From condition (3), we know that $t_1 N \geq \sum_{k=1}^{M} t_k n_k \geq t_M N$. Therefore, as N gets large, the first ratio on the *LHS* converges to 1. We shall maintain this assumption of a large N in the following, so that equilibrium condition (2) can be simplified to

$$\frac{t_i n_i}{n_i - 1} \geq \frac{t_j \left(n_j + 1\right)}{n_j} \tag{6}$$

for all j, and for all i such that $n_i > 1$. These inequalities, together with conditions (3) and (5), define an equilibrium in the second stage.

It can be shown that a second-stage equilibrium satisfying these conditions always exists (refer to Appendix A for a formal proof). Intuitively, the expected payoff for a helper in any faction i is proportional to

$$\lambda_i = \frac{t_i n_i}{n_i - 1}.$$

This is diminishing in the number of helpers in the faction - a larger faction raises the chance of winning, but this effect is more than offset by the reduced share that each helper in the faction can now get. Therefore, even if a faction initially offers higher expected payoff for helpers and attract defectors from other factions, the incentive for further defectors to join will decrease as the faction expands. The opposite is true in factions the ranks of which shrink with defections, as there are now fewer helpers around to split W_2 should the leader win. This closes the gap for profitable deviation. An equilibrium is reached when payoffs for helpers in each faction is no smaller than the maximum payoff from defecting to another faction. Multiple equilibria can arise when some helpers are just indifferent between deviating and not deviating (i.e. when condition (6) is binding for some i and j). Nevertheless, such multiplicity is not robust to small perturbations to the abilities of the faction leaders, giving us uniqueness for almost all values of the players' abilities. We shall assume this to be the case in the remaining parts of this paper.

Given the number and the abilities of the candidates, the equilibrium distribution of helpers across factions has the following property:

Proposition 1: In an equilibrium allocation of helpers, the size of a faction is non-decreasing in the ability of its leader.

Proof: Consider factions i and j and assume that $t_i > t_j$. Condition (6) requires that, in equilibrium,

$$\frac{t_j n_j}{n_j - 1} \geq \frac{t_i(n_i + 1)}{n_i}.$$

Since $t_i > t_j$, this implies

$$\frac{n_j}{n_j - 1} \geq \frac{n_i + 1}{n_i}.$$

Rearranging gives $n_i \geq n_j - 1$. Since both n_i and n_j are integers, $n_i \geq n_j$ whenever $t_i > t_j$. ■

The intuition behind this result is straightforward. If all factions were equal in size, then a candidate of superior ability would have a higher chance of winning because of the "multiplier effect" of the leader's ability implied in equation (2). This would tend to attract defectors from factions headed by less able candidates. Reallocation of helpers will continue until condition (6) holds for all faction, at which point more productive candidates will have attracted a larger number of helpers. With n_i non-decreasing in t_i, it is obvious from equation (2) that a more able candidate will have a better chance of winning. This reinforcement of innate superiority by helping behavior arises from mobility of helpers across factions. The complementarity between the leaders' ability and helping effort produces a positive assortative matching that is standard in the literature (Becker 1973, Kremer 1993, Kremer and Maskin 1996), but contrasts with the results in Chen (2003) and Munster (2004), where the most able candidate tends to attract the most sabotage and may not emerge the most likely winner.

In equilibrium, the number of candidates, M, may exceed the number of factions, L, because some declared candidates may not be able to attract any helper. If a candidate l is unsupported, then condition (6) becomes

$$\lambda_i \geq 2t_l$$

for all i such that $n_i > 1$. Using the notation that $\lambda_{\min} = \min\{\lambda_i | n_i > 1\}$, this can be rewritten as

$$\lambda_{\min} \geq 2t_l. \tag{7}$$

This inequality defines an upper bound for the ability of singleton candidates. If, in equilibrium, this inequality is not satisfied for any candidate, then the contest is played among faction leaders. However, if it holds for some l, since $n_i/(n_i - 1) \leq 2$ for $n_i > 1$, it must be the case that $t_l \leq t_i$. Therefore, "singletons," if they exist, have lower ability than the least able faction leader. If a candidate fails to attract helpers, it is because his chance of winning is too low to offer a competitive expected payoff for potential helpers. In fact, this result can be interpreted as an extension of Proposition 1: less able candidates get fewer helpers, and in the extreme, they may get no helper at all.

The second-stage equilibrium is a function of the number and abilities of the candidates. Any change in these variables will result in realignment of helpers across factions. The identities of the M candidates can be completely described by a profile of their abilities, $t = (t_1, \ldots, t_M)$, and suppose there is an increase in the ability of one candidate i. That is, consider a change from t to t' such that

$$t'_j = \begin{cases} t_j & if \quad j \neq i \\ t'_i > t_i & if \quad j = i \end{cases}$$

The effect on the allocation of helpers can be summarized by the following proposition:

Proposition 2: When the ability of candidate i increases, holding constant the abilities of all other candidates and the number of helpers, then

(i) The size of faction i cannot fall;

(ii) For any $j \neq i$, the size of faction j cannot rise; and

(iii) The minimum payoff to helpers cannot fall.

Proof: See Appendix B.

An exogenous increase in the ability of a candidate raises the expected payoffs for his helpers and attracts defectors from other factions. His faction therefore tend to expand at the expense of other factions. This would increase payoffs for the remaining helpers in other factions in general, and the minimum payoff for helpers in particular. The addition of a new candidate could be thought as an increase in ability of a candidate from zero to some positive number, while a reduction in the number of candidates, a change in the reverse. Therefore, Proposition 2 also applies to the effect of a change in the number of candidates, holding constant the number of helpers. Changes to the abilities of multiple candidates can also be analyzed by repeatedly using the results from an increase in one candidate's ability.[5]

The lineup of candidates is endogenous. It is determined in the first stage of the game when each player, given the anticipated outcome of the second stage allocation as described in this section, decides whether to enter the tournament as a candidate or resign himself to a supporting role.

2. Self-Selection into Candidates and Helpers

In the first stage, each player will choose to be either a candidate or a helper, whichever yields a higher expected payoff in the second-stage equilibrium allocation resulting from his choice, given the choices of other players. However, an equilibrium helper allocation in the second stage can only be defined in terms of the number of helpers in each faction. It does not specify which particular candidate will be helped by each helper. For example, given a set of candidates and a corresponding second-stage equilibrium allocation of helpers in which helper 1 is in faction i while helper 2 is in faction j, switching the

[5] As long as the equilibrium allocation at each step of such an analysis is unique, breaking one change into multiple steps of changes is without loss. In the unlikely case of non-uniqueness, the result characterizes one of the possibly multiple equilibria.

allegiance of helpers 1 and 2 without any further change would also result in a second stage equilibrium. Yet the expected payoffs for helpers are not necessarily the same, as helpers 1 and 2 may be getting different payoffs in these two second-stage equilibria, even if all players have made the same first-stage choices in both cases. Because of the different outcomes to helper 1 in the two cases, his first-stage decision may depend on which assignment of helpers will occur should he become a helper. In general, given a set of candidates, even if the equilibrium helper allocation, $n = (n_1, \ldots , n_M)$, is unique in the second stage, there are still $(N - M)!$ different ways to assign the helpers to achieve the same allocation n. As λ_i will be different for different i, to the players in the first stage, there may be up to $(N - M)!$ equilibrium continuations to the subgame with the same set of candidates. Furthermore, for each possible set of candidates, there are also multiple equilibrium continuations to each subgame with that particular set of candidates. The first stage choice of each player, therefore, depends on the particular combination of equilibrium continuations at each and every subgame. In the following, we shall first characterize the optimal first-stage choice for each player, given a particular combination of equilibrium continuations, before deriving a general characterization of all subgame perfect equilibria of the complete contest.

Where no confusion may arise, let $N = \{1, \ldots , N\}$ be the set of all players. Then a set of candidates can be denoted as $\Gamma \subseteq N$, with cardinality M. Let S be the collection of all possible sets of candidates with at least one candidate and at least one helper. Mathematically, S is the set of all non-empty proper subsets of N.

For any non-empty Γ, let $n(\Gamma) = (n_1(\Gamma), \ldots , n_M(\Gamma))$ be the equilibrium helper allocation in the second stage given Γ, and $\lambda(\Gamma) = (\lambda_1(\Gamma), \ldots , \lambda_M(\Gamma))$ the equilibrium values of λ induced by Γ. Following earlier notations, $\lambda_{\min}(\Gamma)$ represents the minimum $\lambda_i(\Gamma)$ among all factions in the second-stage equilibrium helper allocation.

If there are at least one candidate and at least one helper, we can specify an assignment of helpers to factions such that the total number of helpers in each faction is equal to the equilibrium helper allocation. Formally, for each $\Gamma \in S$, a helper assignment is a function $a^{\Gamma}: N \setminus \Gamma \to \Gamma$ such that

$$\#\{j \in N \setminus \Gamma \mid a^{\Gamma}(j) = i\} = n_i(\Gamma) \qquad \text{for all } i \in \Gamma.$$

In effect, for each helper j, this function assigns to him a candidate i such that the number of helpers each candidate gets is the equilibrium number of helpers in the second stage. In other words, an assignment function specifies the equilibrium continuation to a particular subgame. As we noted earlier, given each Γ, the assignment function is not unique, but for a particular a^{Γ}, helper j will be assigned to faction $a^{\Gamma}(j)$ and get $\lambda_{a^{\Gamma}(j)}(\Gamma)$ accordingly. To avoid notational clutter, we adopt the shorthand $\lambda_{a(j)}(\Gamma) = \lambda_{a^{\Gamma}(j)}(\Gamma)$. In the case where there are no candidates, $\lambda_{a(j)}(\emptyset) = 0$ for all $j \in N$.

Consider any set of candidates Γ. Given the assignment functions for all possible sets of candidates $\{a^s\}_{s\in S}$, Γ is an equilibrium set of candidates if and only if,

$$\frac{t_i n_i(\Gamma)}{\sum_{k\in\Gamma} t_k n_k(\Gamma)} W_1 \geq \frac{\lambda_{a(i)}(\Gamma \setminus \{i\})}{\sum_{k\in\Gamma\setminus\{i\}} t_k n_k(\Gamma \setminus \{i\})} W_2 \qquad \text{for all } i \in \Gamma, \tag{8}$$

and

$$\frac{\lambda_{a(i)}(\Gamma)}{\sum_{k\in\Gamma} t_k n_k(\Gamma)} W_2 \geq \frac{t_i n_i(\Gamma \cup \{i\})}{\sum_{k\in\Gamma\cup\{i\}} t_k n_k(\Gamma \cup \{i\})} W_1 \qquad \text{for all } i \notin \Gamma. \tag{9}$$

Condition (8) requires that, for any candidate, the payoff from deviating to become a helper, given the assignment of helpers after his deviation, is lower than his equilibrium payoff as a candidate. Similarly, condition (9) suggests that, for any helper, given the current helper assignment, it is not profitable to deviate and become a candidate. The two conditions together imply that no player could find unilateral profitable deviations given the strategy of all other players and the specified equilibrium continuation to the subgame should they deviate. Therefore, any (pure strategy) subgame perfect equilibrium of the whole contest can be characterized by a profile of assignment functions and a partition of the player, $(\{a^s\}_{s\in S}, \Gamma)$, such that given $\{a^s\}_{s\in S}$, Γ satisfies the system of inequalities given by (8) and (9).

It is obvious that there can be multiple subgame perfect equilibria. Theoretically, we can select a possible combination of the assignment functions, $\{a^s\}_{s\in S}$, go through all $\Gamma \subseteq N$ and check whether conditions (8) and (9) are satisfied for each, hence finding all equilibrium sets of candidates given this particular combination of assignment functions. Repeating this exercise for each and every possible combination of assignment functions, we can find all subgame perfect equilibria of the contest. Nevertheless, doing so analytically is a tedious exercise that yields few insights. So, instead, we shall focus on a class of subgame perfect equilibria in which all candidates are more able than all helpers. In other words, if we order the players such that $t_1 \geq t_2 \geq \ldots \geq t_N$, then in this class of equilibrium, the first M players will become candidates while the remaining will be helpers. Since players are sorted into the leading and supporting roles perfectly by their ability in this class of equilibrium, we call it a *Perfect Sorting Equilibrium*. Some results on this class of equilibrium can be summarized as follows:

Proposition 3: Given the set of players and the reward structure, there always exists a perfect sorting equilibrium that is subgame perfect. The number of candidates in such an equilibrium can range from 1 (everybody belongs to a grand coalition) to N (everyone is a candidate). More precisely, there always exists a natural number M, $1 \leq M \leq N$, and a combination of assignment functions $\{a^s\}_{s\in S}$ such that $(\{a^s\}_{s\in S}, \Gamma)$ satisfies conditions (8) and (9) for $\Gamma = \{1, \ldots, M\}$.

Proof: See Appendix C.

Under the assumptions of our model, the intuition behind perfect sorting is obvious. Because of the multiplier effect of a leader's ability, more productive players have an advantage as candidates, while less able players would benefit more from supporting roles because of the homogeneity of helping efforts. The existence of a perfect sorting equilibrium, however, does not preclude the possibility of equilibria in which some helpers are superior in ability to some candidates. Imperfect sorting arises mainly from a coordination problem among players. Even if there is no coordination issue in the second stage (for example, if the difference in payoffs across factions is insignificant), it is still possible that sorting is imperfect in the first stage. With some equilibrium sets of candidates, helper i may be superior in ability to but enjoys a lower expected payoff than candidate j, because i's entry into the contest may lower the payoffs for all candidates (including i's) by so much that i is better off not deviating from his initial helping role. Thus, even if there is a joint deviation by players i and j (e.g. the players switching roles) that would not make either of them worse off, it cannot happen in our setup, as only unilateral decisions are allowed. This contrasts our result with the usual coalition formation literature in cooperative game theory, in which a subset of players can jointly improve their payoffs as long as none of them are made worse off (e.g.: Chatterjee et al. 1993; Perry and Reny 1994).

Despite the possibility of imperfect sorting equilibria, we shall focus, from now on, on the more interesting case of a perfect sorting equilibrium and explore its comparative statics in the next section.

C. Comparative Statics in a Perfect Sorting Equilibrium

In the sorting equilibrium described in the previous section, the candidates are always the most able players. Therefore, given the distribution of the players' abilities, the characteristics of the candidates are completely described by the number of candidates M.

The equilibrium expected payoffs for helpers of these candidates need not be equal because of discreteness in the number of faction members. Nevertheless, keeping track of the inequalities at the margin complicates derivations without offering any additional insight. Accordingly, in the following, we shall simplify the analysis by assuming exact equalization of expected payoffs for helpers across factions in an equilibrium, so that

$$\frac{t_i n_i(M)}{n_i(M)-1} = \frac{t_j n_j(M)}{n_j(M)-1} = \lambda(M) \tag{10}$$

for all i, j such that $n_i, n_j > 1$. Condition (7) for singleton candidates can now be rewritten as

$$t_l \leq \frac{\lambda(M)}{2} \tag{11}$$

and an equilibrium allocation of helpers can then be characterized by equations (10), (11) and (5). Also, in a perfect sorting equilibrium, conditions (8) and (9) together imply that for $1 < M < N$,

$$\xi(M+1) \leq \frac{W_2}{W_1} \leq \xi(M) \tag{12}$$

where

$$\xi(M) = \frac{t_M n_M(M)}{\sum_{k=1}^{M} t_k n_k(M)} \frac{\sum_{k=1}^{M-1} t_k n_k(M-1)}{\lambda(M-1)}.$$

$\xi(M)$ represents the chance for the marginal player of securing rewards as a candidate relative to his chance as a helper, weighted by the share that he can get in the case of success. It reduces to $(n_M(M) - 1)(N - M + 1)/(N - M)$ when there are no singleton candidates. Equation (12) shows that in a perfect sorting equilibrium, $\xi(M)$ must be non-increasing locally in M so that the incentive to contest rather than help tend to increase with ability. These equations will be used to derive the comparative static results on the effects of exogenous changes in the structure of the tournament and in the abilities of contestants on a perfect sorting equilibrium.

1. A Change in the Reward Structure

For simplicity, assume that equation (12) is binding in an initial equilibrium with M contestants, so that

$$\xi(M) = \frac{W_2}{W_1}, \tag{13}$$

where $\Delta\xi(M)/\Delta M \leq 0$. It then follows that, in a perfect sorting equilibrium,

$$\frac{\Delta M}{\Delta W_1} = -\frac{W_2}{W_1^2} \bigg/ \frac{\Delta\xi}{\Delta M} > 0 .$$

The incentive effect of a change in the top prize is obvious: a larger W_1 is going to encourage more players to become candidates, leaving fewer available helpers. The resulting reduction in faction sizes increases the equilibrium payoffs for helpers and therefore λ. This also raises the upper bound for singleton candidates (equation (11)), which implies that more candidates will find it difficult to attract the fewer helping hands around, and some may end up not having any at all. Hence, an increase in W_1 is going to increase the number of candidates but reduce the size of incumbent factions, while increasing the likelihood or the number of unassisted candidacies. In the extreme case, when W_1 is very large, everyone becomes singleton candidates.

Because W_2 enters into the equilibrium conditions only as a ratio of W_1, its effect on the equilibrium factional structure is exactly the opposite of that of W_1:

$$\frac{\Delta M}{\Delta W_2} = \frac{1}{W_1} \bigg/ \frac{\Delta\xi}{\Delta M} < 0$$

Thus, an increase in the "second prize" will encourage more workers to become helpers, reduce the number of contestants (particularly of singleton candidates), but raise the number and the size of factions. As W_2 converges towards W_1, only the most able worker will have the incentive to go for the top prize, and everyone else plays a supporting role in a grand coalition headed by the top player in a non-contest.

2. A Change in the Spread of Abilities

The effect of a more lopsided contest on the equilibrium factional structure can be analyzed by extending the spread of the abilities of the workers. Consider equal and opposite changes in the abilities of two players p and q, so that $\Delta t_p = -\Delta t_q > 0$. Suppose $t_p > t_q$, so that the changes leave the average ability of all workers unchanged but increase the variance of the abilities. The effect of such a change will depend on whether p and q are leaders or helpers in the initial equilibrium. There are three different cases.

a. Both p and q are helpers or singleton candidates

If both p and q are helpers or singletons, then as long as the changes are not large enough to change their status, the equilibrium will not be perturbed since the factional structure depends only on the abilities of leaders.

b. p is a leader and q is a helper (or singleton candidate)

As in the first case, the change in t_q has no effect, but an increase in t_p will cause a reshuffling of helpers across factions in the second stage of the game. Using equation (10), the size of any faction n_k can be expressed as a function of an arbitrary n_i, for $i \in [1,L]$ and $L \leq M$. Substituting into equation (7) gives

$$\sum_{k=1}^{L} \frac{t_k(n_i - 1)}{t_i n_i - t_k(n_i - 1)} = N - M, \tag{14}$$

which implies that

$$\frac{\Delta n_i}{\Delta t_p} = -\frac{n_i\left(n_i'-1\right)n_p n_p'}{\sum_{k=1}^{L} t_k n_k n_k'} < 0 \qquad (15)$$

$\forall\, i \in [1,L]$, $L \leq M$ and $i \neq p$, and

$$\frac{\Delta n_p}{\Delta t_p} = \frac{\sum_{k=1}^{L} n_k'\left(n_k - 1\right)/\,t_p'}{\left(\sum_{k=1}^{L} n_k'\left(n_k - 1\right)/\,n_p'\left(n_p - 1\right)\right) - 1} > 0 \quad , \qquad (16)$$

where n and n' represent respectively faction sizes before and after the change in t_p. In other words, holding M constant, an increase in the ability of a faction leader is going to boost support for his candidacy at the expense of all others. But as the number of helpers in other factions drops, λ increases and changes decisions in the first stage. Equation (13) implies that

$$\frac{\Delta M}{\Delta t_p} = -\frac{\Delta \xi}{\Delta t_p}\bigg/\frac{\Delta \xi}{\Delta M}\,.$$

It can be shown that $\Delta\xi/\Delta t_p < 0$ (see Appendix D). Given (12), this implies that $\Delta M/\Delta t_p < 0$. Intuitively, the rise in λ signals a higher payoff for helpers, resulting in a reduction in the number of candidates. The effect on the number of singleton candidates is, however, ambiguous, as the upper bound for the ability of these candidates (defined by equation (11)) also increases with λ. Nevertheless, a more uneven distribution of abilities will tend to discourage competition both by reducing the number of candidates and by redistributing helpers in favour of the strengthened candidate. In the extreme case, when $p = 1$ and t_1 becomes very large, the runaway favorite siphons off support from all potential challengers, resulting in a grand coalition.

c. Both p and q are faction leaders

In this case, the effect of equal and opposite changes in t_p and t_q ($\Delta t_p > 0$) on the size of each faction in the second stage allocation is the sum of two separate effects. Using equations (15) and (16), the effect on n_p can be written as:

$$\frac{\Delta n_p}{\Delta t_p}\Delta t_p + \frac{\Delta n_p}{\Delta t_q}\Delta t_q = \left(\frac{\Delta n_p}{\Delta t_p} - \frac{\Delta n_p}{\Delta t_q}\right)\Delta t_p ,$$

while the effect on n_q is

$$\frac{\Delta n_q}{\Delta t_p}\Delta t_p + \frac{\Delta n_q}{\Delta t_q}\Delta t_q = \left(\frac{\Delta n_q}{\Delta t_p} - \frac{\Delta n_q}{\Delta t_q}\right)\Delta t_p .$$

In both cases, the two effects are reinforcing, causing the strengthened candidate's faction to expand and the weakened candidate's faction to contract. The effect on n_i ($i \neq p,q$) can be expanded using equation (15):

$$\frac{\Delta n_i}{\Delta t_p}\Delta t_p + \frac{\Delta n_i}{\Delta t_q}\Delta t_q = \frac{n_i\left(n_i'-1\right)}{\sum_{k=1}^{L} t_k n_k n_k'}\left(n_q n_q' - n_p n_p'\right) .$$

Since $n_p > n_q$ and $n_p' > n_q'$, the net effect is negative. In this case, the two effects of the changes in t_p and t_q on the size of any other faction are offsetting, with the faction losing support to p but gaining helpers from q. The former effect dominates, however, and faction p emerges as the only faction to gain strength while all others (and faction q in particular) shrink, given the total number of helpers. Therefore, the qualitative result is the same as in case b above, and the same analysis of first stage decisions on candidacy applies, with $\Delta M / \Delta t_p < 0$ and a reduction in the number of candidates in a more lopsided contest.

These results indicate that a more skewed distribution of the abilities of leaders will, in general, result in fewer factions and less competition. Having a frontrunner (or frontrunners) in a contest will therefore facilitate consolidation of support as helpers gravitate towards this contestant(s), and marginal candidates may no longer find it worthwhile to fight a losing cause. This is perhaps most vividly illustrated in electoral politics. In U.S. presidential primaries, the fates of candidates are often sealed by elections in a few crucial states. Those who fall behind at these junctures will face dwindling resources that will often force them to withdraw, while the candidacies of frontrunners will gain further strength by absorbing the support for failed candidates. In fact, unity is important at the end of a primary contest as the party prepares itself for the external challenge that follows. It is therefore not coincidental that incentives are designed so that a primary race is rarely taken all the way to the party convention which, more often than not, is staged as a coronation rather than a final showdown.

Whether in a political election or a promotion tournament within a firm, the absence of frontrunners or strong candidates can have significant implications both before and after a contest. Apart from compromised performance during the contest due to the lack of cooperation among players, the winning candidate can also be hampered by the dearth of talent in his team after the dust settles. This is because a crowded field with no favourites reduces the number as well as the quality of helpers. Whoever wins the contest will then have to appoint mediocre supporters to senior administrative or executive positions. It is therefore not coincidental that fragmented political systems with no dominant parties are often perenially plagued by poor governance that further contributes to instability and stagnation.

III. Discussion and Conclusion

In this paper, we have presented a model for help and factionalism in politics and within organizations. It is shown that if the winner of a contest is given control over resources that he can use to reward supporters, help among players is encouraged as factions form around candidates. The sorting into leading and supporting roles is, however, not random, as helpers are generally inferior in abilities to leaders. It is also shown that more able leaders will always attract more helpers in equilibrium, and therefore stand a better chance of winning the contest. The least able candidates may not enjoy much support and may even be left to fend for themselves in the contest.

The number of candidates and the size of factions are also affected by the reward structure. A larger top prize relative to the "second prize" that is to be distributed among supporters of the winner will result in more factions and less cooperation among players, while the opposite is true if the second prize is large. This implies that by manipulating these rewards, a firm can induce as much or as little help among its workers as needed. A more decentralized organizational structure, in which division or team leaders are allowed greater discretion in personnel or resource allocation decisions, may encourage teamwork and cooperation within the division. There may be less industrial politics at the top levels if the CEO is given a freer hand in appointing his top assistants, not just because of better working relationship between the CEO and his own appointees after his promotion, but also because of less factionalism and greater cooperation within factions in the competition for the position. Lazear (1989) also observes that in a firm organized by products, the CEO is often chosen from among heads of different product divisions, because it is important to maintain cooperation among colleagues working on the same product line. Extending that argument, our analysis suggests that there tends to be more "cronyism" in firms with such organizational structures, because having more resources for rewarding supporters would facilitate teamwork within divisions.

The reward structure may, however, also depend on the technology of production as suggested by Itoh (1991). Even if we abstract from the incentive effects of a compensation scheme on workers' efforts, it is still not the case that less competition and greater cooperation is always to be preferred. In some creative industries, diversity is a very important part of the production process and competing ideas should be encouraged rather than restrained. This is particularly the case if team production is subject to high diminishing returns and cooperation among workers does not result in substantial gain in team productivity. Thus, a disproportionate reward for the top performer may provide the right incentive in architectural firms and haute couture houses where the competing teams are relatively small. In the extreme case, when there is no complementarity in different workers' inputs, it may be more efficient for the firm to offer a simple tournament scheme with no payoff for helping.

Much has been said about differences in corporate cultures across firms. Some firms thrive on a competitive environment that encourages a rivalrous relationship among workers in order to bring out their best efforts. Others value camaraderie and congenial relationship among colleagues that foster cooperation. The contrast extends to international comparisons as well. It is often suggested that the team spirit is emphasized in Japanese corporations while individual achievements and innovations are more prized in western, particularly U.S., corporate cultures. Ultimately, such differences across firms and countries may be just a reflection of differences in incentive schemes dictated by the underlying management philosophies and the choice of production techniques, certain aspects of which have been discussed above.

Although the proposed relationship between the workers' helping behavior and the firm's compensation scheme is intuitive, it is difficult to find empirical support for the hypotheses from corporate data, as some key variables (such as the "second prize" to be shared among helpers, the size of factions, and the number of candidates) are not readily observable. However, the application of our analysis to political elections suggests a more ready source of data. Just like an executive climbing the corporate ladder, an aspiring politician faces the same dilemma of whether to present himself as a candidate in an election or play a supportive role in the hope of earning a future political appointment or other favors. Altering the structure of rewards can therefore profoundly affect participation in the electoral process and the makeup of party electoral politics. With different offices offering different resources directly and indirectly available to winning candidates, often clearly defined in statutory terms, and campaign organization and funding potentially observable, political elections in different jurisdictions present a natural experiment for testing the relationship between reward structure and helping behavior. This also suggests that, whether in the corporate boardroom or the corridors of power in the government, our model can offer potentially interesting and useful insights into designing the right incentive mechanism.

Appendix

A. Second Stage Equilibrium

In this appendix, we will construct an algorithm to show the existence and generic uniqueness of a second stage equilibrium helper allocation.

Suppose there are M candidates. An allocation of helper is an M-tupled $n = (n_1,..., n_2)$ satisfying

$$n_i \geq 1 \qquad \text{for all } i = 1, \dots , M,$$

and

$$\sum_{i=1}^{N} n_i = N\,.$$

The algorithm goes in rounds. Denote the allocation of helpers in round r as $n^r = (n_1^r,..., n_M^r)$. In any round, given n^r, define

$$\lambda_i^r = \begin{cases} \dfrac{t_i n_i^r}{n_i^r - 1} & if \quad n_i^r \geq 1 \\ \\ +\infty & if \quad n_i^r = 1 \end{cases}$$

and

$$\hat{\lambda}_i^r = \frac{t_i\left(n_i^r + 1\right)}{n_i^r}\,.$$

Then look for

$$j(r) = \arg\min_{i=1,\dots,M} \lambda_i^r\,,$$

$$k(r) = \arg\max_{i=1,\dots,M} \hat{\lambda}_i^r$$

Ties, if any, are broken lexicographically. The algorithm stops at round r if $\lambda_{j(r)}^r \geq \hat{\lambda}_{k(r)}^r$.

If, instead, $\lambda_{j(r)}^r < \hat{\lambda}_{k(r)}^r$, define

$$n_i^{r+1} = \begin{cases} n_i^r - 1 & if \quad i = j(r) \\ n_i^r + 1 & if \quad i = k(r) \\ n_i^r & otherwise \end{cases}$$

and continue to run the algorithm.

The idea of the algorithm is simple: at each round, given an allocation of helping effort, we look for the lowest current payoff to a helper and the highest possible payoff from a unilateral deviation. If the latter is strictly higher than the former, we move one helper from the faction that is currently offering the lowest payoff to the faction that is offering the highest for a defector. Notice that, as $\lambda_i^r \geq \hat{\lambda}_i^r$ for all i and for all r, whenever there is a move it must involve two factions, so the "deviation" in the algorithm is well-defined.

This algorithm has two useful properties:

Lemma 1: If a helper joins faction i at some round $r > 0$, then for any later rounds, no helper will leave faction i. More precisely, if $n_i^{r+1} = n_i^r + 1$, then $n_i^{\rho+1} \geq n_i^\rho$ for all $\rho > r$.

Proof: Suppose not. Then there is a round $\rho > r$ such that $n_i^{\rho+1} < n_i^\rho$. Further assume that $n_i^\rho = n_i^{r+1}$, that is, round ρ is the first time after r that the number of helpers in faction i has changed. (This is without loss of generality, since we can pick ρ to be the first round after r that a helper leaves faction i. If some other helpers have joined faction i between round r and ρ, call the last round before ρ in which this happens round r' and use r' in place of r for the analysis.) Suppose at round ρ, faction i loses a helper to faction j. This means that

$$\frac{t_i n_i^\rho}{n_i^\rho - 1} = \frac{t_i\left(n_i^r + 1\right)}{n_i^r} < \frac{t_j\left(n_j^\rho + 1\right)}{n_j^\rho} \tag{A1}$$

Recall that in round r a helper joins faction i. This suggests

$$\frac{t_i\left(n_i^r + 1\right)}{n_i^r} \geq \frac{t_j\left(n_j^r + 1\right)}{n_j^r}$$

These two inequalities together implies that $n_j^\rho < n_j^r$, meaning that faction j must have lost at least one helpers between round r and ρ. Thus, there exists an s such that $r < s < \rho$ and $n_j^\rho = n_j^s$ - 1. However, at round s, condition (A1) requires

$$\lambda_i^s = \frac{t_i\left(n_i^r + 1\right)}{n_i^r} < \frac{t_j\left(n_j^\rho + 1\right)}{n_{ij}^\rho} = \frac{t_j n_j^s}{n_j^s - 1} = \lambda_j^s$$

which means that faction j could not have lost a helper in round s. This is a contradiction. ■

Lemma 2: If a helper leaves faction i at some round $r > 0$, then no helper will join faction i in any subsequent round. More precisely, if if $n_i^{r+1} = n_i^r$ - 1, then $n_i^{\rho+1} \le n_i^\rho$ for all $\rho > r$.

Proof: Omitted. This is simply reversing the direction of the proof of Lemma 1.

An immediate consequence of these two properties is that, given any set of candidates and any initial allocation of helpers n^0 such that $n_i^0 \ge 1$ for all i and $\sum_{i=1}^M n_i = N$, the algorithm we described must stop in some finite time. This is because the size of a faction can only change monotonically as the algorithm runs, and there are only finitely many helpers to be added to or removed from a faction.

For the existence result, it remains to show that when the algorithm stops at round r, the helper allocation n^r is indeed an allocation of helpers satisfying the equilibrium conditions of the second stage problem. Recall that the second stage equilibrium helper allocation requires

$n_i \ge 1$ for all i,

$\lambda_i \ge \hat{\lambda}_j$ for all j such that $n_j > 1$, for all i,

and $$\sum_{i=1}^M n_i = N .$$

The first and the third conditions are trivially satisfied by construction. Notice that the algorithm stops at round r only if

$$\min_i \lambda_i^r \geq \max_i \hat{\lambda}_i^r ,$$

meaning that the second condition is also satisfied. Therefore, n^r is indeed an equilibrium helper allocation. As the algorithm always stops at some finite time and gives n for any set of candidates and any initial helper allocation, a second stage equilibrium helper allocation always exists.

For the uniqueness result, suppose there are two equilibrium helper allocations, n and n', given a set of candidates. If $n \neq n'$, then there exists i, j such that $n_i > n_i'$ and $n_j < n_j'$. Since n is an equilibrium helper allocation,

$$\frac{t_i n_i}{n_i - 1} \geq \frac{t_j\left(n_j + 1\right)}{n_j} \geq \frac{t_j n_j'}{n_j' - 1}.$$

Similarly, as n' is an equilibrium helper allocation,

$$\frac{t_j n_j'}{n_j' - 1} \geq \frac{t_i\left(n_i' + 1\right)}{n_i'} \geq \frac{t_i n_i}{n_i - 1}.$$

These inequalities can only be true if they all hold as strict equality. Therefore, we have

$$\lambda_i = \hat{\lambda}_j = \lambda_j' = \hat{\lambda}_i'.$$

But this implies that, in the n equilibrium, helpers in faction i is just indifferent between staying and deviating to faction j, while in the n' equilibrium, helpers in faction j are indifferent between stay and deviating to faction i. Thus, multiple equilibria can only result from indifference at the margin. Moreover, if we perturb t_i to $\tilde{t}_i = t_i + \varepsilon$ for any $\varepsilon > 0$, then

$$\tilde{\lambda}_i > \lambda_i = \lambda_j'.$$

and there can no longer be multiple equilibria (between factions i and j). Similarly, if we perturb t_i to $\tilde{t}_i = t_i - \varepsilon$, there will be no multiple equilibria either. Since the multiplicity of equilibria is not robust to any small perturbation to the ability values, we may say that the second stage equilibrium helper allocation is generically unique. More precisely, if we draw the profile of ability according to some smooth, continuous distribution, we can draw an ability profile that gives a unique second stage equilibrium with probability 1.

B. Proof of Proposition 2

Proposition 2: When the ability of candidate *i* increases, holding the ability of all other candidates and the number of helpers unchanged, then

(i) The size of faction *i* cannot fall;

(ii) For any $j \neq i$, the size of faction *j* cannot rise; and

(iii) The minimum payoff to helper (and therefore $\lambda_{\min}$) cannot fall.

Proof: Denote the original ability profile as $t = (t_1, \dots, t_M)$ and the new one as $t' = (t_1', \dots t_M')$. Write the original equilibrium allocation as *n*. We will use the original equilibrium as the initial allocation and run the algorithm constructed in Appendix A to obtain the new equilibrium allocation, .

To show (i), consider the initial condition of the algorithm. Since

$$\hat{\lambda}_i^0 = \frac{t_i'(n_i + 1)}{n_i} > \hat{\lambda}_i$$

while $\lambda_j^0 = \lambda_j$ and $\hat{\lambda}_j^0 = \hat{\lambda}_j$ for all $j \neq i$, if there is an adjustment in the first round, it must involve adding a helper to faction *i*. By Lemma 1, faction *i* cannot lose any helper in the later rounds of the algorithm, so $n_i' > n_i$. If instead there is no adjustment in the first round, $n' = n$ and $n_i' = n_i$.

To prove (ii), suppose not and there is a faction $j \neq i$ with $n_j' > n_j$. Let the first additional helper to faction *j* come from faction *k* in round *r*. This means that

$$\frac{t_j(n_j + 1)}{n_j} \geq \frac{t_k' n_k^r}{n_k^r - 1}.$$

Since n is an equilibrium,

$$\frac{t_j(n_j+1)}{n_j} \le \frac{t_k n_k}{n_k - 1}.$$

Since $t_k' \ge t_k$, these two inequalities together implies $n_k^r > n_k$. But this means that faction k must have gained at least one helper before round r. By Lemma 1, it cannot lose a helper to faction j. This is a contradiction.

To see (iii), notice first of all that, if there is no change to the equilibrium helper allocation, the statement follows immediately. Now suppose $n' \neq n$. Let r be the last round of adjustment before the algorithm stops, and, following (i) and (ii) above, a helper must have moved from some faction j to i in that round. This means

$$\lambda_i' = \hat{\lambda}_i^r > \lambda_i^r \ge \lambda_j \ge \lambda_{\min}.$$

At the same time, by (ii) no faction other than i can gain helpers. Thus

$$\lambda_k' \ge \lambda_k \ge \lambda_{\min} \qquad \text{for all } k \neq i.$$

Therefore $\lambda_k' \ge \lambda_{\min}$ for all $k = 1, \ldots, M$ and (iii) follows. ■

C. Existence of Perfect Sorting Equilibrium

In order to simplify notations, the following shorthand will be used in this Appendix:

$$n_i = n_i(\Gamma),$$

$$n_k^i = n_k(\Gamma \cup \{i\}),$$

$$n_k^{-i} = n_k(\Gamma \setminus \{i\}).$$

If we order all players by their ability so that $t_1 \ge t_2 \ge \ldots \ge t_N$, then a perfect sorting equilibrium is given by $M = \{1, \ldots, M\}$ and a combination of assignment functions $\{a^s\}_{s \in S}$ such that

$$\frac{t_i n_i(M)}{\sum_{k=1}^{M} t_k n_k(M)} W_1 \geq \frac{\lambda_{a(i)}(M \setminus \{i\})}{\sum_{k \in M \setminus \{i\}} t_k n_k(M \setminus \{i\})} W_2 \qquad \text{for all } i \leq M; \tag{C1}$$

and

$$\frac{\lambda_{a(j)}(M)}{\sum_{k=1}^{M} t_k n_k(M)} W_2 \geq \frac{t_j n_j(M \cup \{j\})}{\sum_{k \in M \cup \{j\}} t_k n_k(M \cup \{j\})} W_1 \qquad \text{for all } j > M. \tag{C2}$$

Pick a combination of assignment functions which, for any $s \in S$, $a^s(j)$ is such that $\lambda_{a(j)}(s)$ is increasing in j. In other words, given any set of candidates, the assignment will be such that the least able helper will always get the highest λ, while the most able helper will always get $\lambda_{\min}$. This kind of assignment is always possible so our combination of assignment functions is well-defined. If, with these assignment functions, we can always find an M that satisfies conditions (C1) and (C2), then a perfect sorting equilibrium always exists.

For an arbitrary M such that $1 \leq M \leq N$, the following lemmas can be established.

Lemma 3: Given M and the assignment functions we have picked, if the most able helper does not want to deviate to become a candidate, no helper would like to deviate.

Proof: Consider condition (C2). Let $i = M + 1$ be the most able helper. By definition, $t_i > t_j$ for all $j > M$. Our assignment function also specifies $\lambda_{a(j)}(M) = \lambda_{\min}(M)$. Hence, for any $j > M$, the LHS of (C2) must be larger than that for i. To prove the lemma, it suffices to show that the RHS of (C2) is increasing in ability.

The candidate sets $M \cup \{i\}$ and $M \cup \{j\}$ differ only by the ability of one candidate, so Proposition 2 applies. This gives $n_k^i \leq n_k^j$ for all $k \leq M$ and $n_i^i \geq n_j^j$. We would like to look for the sign of

$$\frac{t_i n_i^i}{\sum_{k \in M \cup \{i\}} t_k n_k^i} - \frac{t_j n_j^j}{\sum_{k \in M \cup \{j\}} t_k n_k^j},$$

which is the same as

$$t_i n_i^i \left(\sum_{k \in M} t_k n_k^j + t_j n_j^j \right) - t_j n_j^j \left(\sum_{k \in M} t_k n_k^i + t_i n_i^i \right)$$

$$= \sum_{k \in M} \left(t_i n_i^i n_k^j - t_j n_j^j n_k^i \right) > 0$$

Thus the RHS of (C2) is indeed increasing in t_i, which gives the lemma. ■

Lemma 4: Given M and the assignment functions we have picked, if the least able candidate does not want to deviate and become a helper, no candidate would like to deviate.

Proof: Given the assignment function, any candidates who deviate from a perfectly sorted partition must get $\lambda_{\min}$ after deviation. Then condition (C1) can be written as

$$\frac{W_1}{t_i n_i \sum_{k \in M} t_k n_k(M)} \geq \frac{\lambda_{\min}\left(M \setminus \{i\}\right)}{t_i n_i(M) \sum_{k \in M \setminus \{i\}} t_k n_k\left(M \setminus \{i\}\right)} W_2 .$$

Given M, the LHS of this inequality is constant for all $i \leq M$. Let $j = M$ be the least able candidate. We need to show that, for any $i \leq M$ that $t_i > t_j$,

$$\frac{\lambda_{\min}\left(M \setminus \{i\}\right)}{t_i n_i \sum_{k \in M \setminus \{i\}} t_k n_k^{-i}} \leq \frac{\lambda_{\min}\left(M \setminus \{j\}\right)}{t_j n_j \sum_{k \in M \setminus \{j\}} t_k n_k^{-j}} . \tag{C3}$$

If $M = 1$ the statement is trivial. If $M = 2$, then

$$\frac{\lambda_{\min}\left(M \setminus \{i\}\right)}{\sum_{k \in M \setminus \{i\}} t_k n_k^{-i}} = \frac{1}{N-1} = \frac{\lambda_{\min}\left(M \setminus \{j\}\right)}{\sum_{k \in M \setminus \{j\}} t_k n_k^{-j}} .$$

Since $t_i n_i > t_j n_j$, inequality (C3) holds.

Now suppose $M \geq 3$. Note that $M \setminus \{i\}$ and $M \setminus \{j\}$ differ only by the ability of one candidate – candidate j in $M \setminus \{i\}$ is less able than candidate i in $M \setminus \{j\}$. Proposition 2 applies and we have

$$\lambda_{\min}(M\setminus\{i\}) \le \lambda_{\min}(M\setminus\{j\}),$$

$$n_k^{-i} \ge n_k^{-j} \text{ for all } k \ne i, j, k \in M,$$

$$n_j^{-i} \le n_i^{-j}.$$

In addition, since the total number of players is unchanged,

$$\sum_{\substack{k\ne i,j \\ k\in M}} n_k^{-j} + n_i^{-j} = \sum_{\substack{k\ne i,j \\ k\in M}} n_k^{-i} + n_j^{-i}$$

$$n_i^{-j} - n_j^{-i} = \sum_{k\ne i,j}\left(n_k^{-i} - n_k^{-j}\right) \ge 0.$$

Consider then

$$t_i n_i \sum_{k\in\Gamma\setminus\{i\}} t_k n_k^{-i} - t_j n_j \sum_{k\in\Gamma\setminus\{i\}} t_k n_k^{-i}$$
$$= t_i n_i\left(\sum_{k\ne i,j} t_k n_k^{-i} - \sum_{k\ne i,j} t_k n_k^{-j} + \sum_{k\ne i,j} t_k n_k^{-j} + t_j n_j^{-i}\right) - t_j n_j\left(\sum_{k\ne i,j} t_k n_k^{-j} + t_i n_i^{-j}\right) \quad \text{(C4)}$$
$$= t_i n_i \sum_{k\ne i,j} t_k\left(n_k^{-i} - n_k^{-j}\right) + \left(t_i n_i - t_j n_j\right)\sum_{k\ne i,j} t_k n_k^{-j} + t_i t_j\left(n_i n_j^{-i} - n_j n_i^{-j}\right)$$

Note that the first and the second term are both (weakly) positive. If $n_i^{-j} - n_j^{-i} = 0$, the last term will also be weakly positive so the whole expression must be positive. If instead $n_i^{-j} - n_j^{-i} > 0$, we can define

$$\bar{t}_k = \sum_{k\ne i,j} t_k \frac{n_k^{-i} - n_k^{-j}}{\sum_{k\ne i,j}\left(n_k^{-i} - n_k^{-j}\right)} = \frac{1}{n_i^{-j} - n_j^{-i}} \sum_{k\ne i,j} t_k\left(n_k^{-i} - n_k^{-j}\right)$$

as the weighted average of the ability of all faction leaders $k \in M$, $k \ne i, j$ who have lost helpers as the set of candidates change from $M\setminus\{i\}$ to $M\setminus\{j\}$. Since all $k \le M$ and j is the least able candidate in M, $\bar{t}_k \ge t_j$. We can then rewrite the expression (C4) as

$$t_i n_i \bar{t}_k \left(n_i^{-j} - n_j^{-i}\right) + \left(t_i n_i - t_j n_j\right) \sum_{k \neq i,j} t_k n_k^{-j} - t_i t_j n_j \left(n_i^{-j} - n_j^{-i}\right) + t_i t_j n_j^{-i} \left(n_i - n_j\right)$$
$$\geq t_i \left(n_i^{-j} - n_j^{-i}\right)\left(\bar{t}_k n_i - t_j n_j\right) \geq 0.$$

This means that the denominator of the LHS of (C3) is bigger than that on the RHS. At the same time, since $\lambda_{\min}(M \setminus \{i\}) \leq \lambda_{\min}(M \setminus \{j\})$,, the numerator of the LHS of (C3) is smaller than that on the RHS. Therefore, inequality (C3) must hold. This completes the proof.■

Using these two lemmas, inequalities (C1) and (C2) reduce to

$$\frac{t_M n_M(M)}{\sum_{k=1}^{M} t_k n_k(M)} W_1 \geq \frac{\lambda_{\min}(M-1)}{\sum_{k=1}^{M-1} t_k n_k(M-1)} W_2;$$

and

$$\frac{\lambda_{\min}(M)}{\sum_{k=1}^{M} t_k n_k(M)} W_2 \geq \frac{t_{M+1} n_{M+1}(M+1)}{\sum_{k=1}^{M+1} t_k n_k(M+1)} W_1.$$

In the case of $M = N$, define $t_{N+1} = 0$. To simplify notations, let

$$C(M) = \frac{t_M n_M(M)}{\sum_{k=1}^{M} t_k n_k} W_1$$

and

$$H(M) = \frac{\lambda_{\min}(M-1)}{\sum_{k=1}^{M-1} t_k n_k(M-1)} W_2.$$

Then, a perfect sorting equilibrium exists if we can find an M, $1 \leq M \leq N$, such that

$$C(M) \geq H(M), \quad \text{(C5)}$$

and

$$C(M+1) \leq H(M+1). \quad \text{(C6)}$$

Note that

$$C(1) = W_1 > 0 = H(1),$$

and

$$C(N+1) = 0 < W_2 = H(N+1).$$

Then, there exists some $m \in \{1, ..., N+1\}$ such that $C(m) < H(m)$. Denote the smallest of these m's, m^*. Since $C(1) > H(1)$, $m^* \geq 2$. Now let $M = m^*$-1 and M satisfies (C5) and (C6), as well as $1 \leq M \leq N$. This M, together with $\{a^s\}_{s \in S}$, the combination of assignment functions we have picked, constitute a subgame perfect equilibrium. Therefore, a perfect sorting equilibrium always exists. It should also be noted that our choice of assignment functions is only sufficient but not necessary for the existence of a perfect sorting equilibrium. Within a more specific setting, it is possible to construct a perfect sorting equilibrium with some other assignment functions.

D. $\Delta\xi/\Delta t_p < 0$

If there are no singleton candidates ($L = M$), then

$$\sum_{k=1}^{M} t_k n_k(M) = \lambda(M) \sum_{k=1}^{M} (n_k(M) - 1) = \lambda(M)(N - M)$$

and equation (10) can be written as

$$\xi(M) = (n_M(M) - 1)\frac{N - M + 1}{N - M} = \frac{W_2}{W_1}.$$

$\Delta n_M/\Delta t_p < 0$ by equation (11), therefore, $\Delta\xi(M)/\Delta t_p < 0$.

If there are singleton candidates ($L < M$), then when the Mth candidate switches to helping, he will either (i) join a faction and help an existing leader; or (ii) help the highest ability singleton candidate. Suppose the marginal candidate switches to helping leader j. This implies that none of the helpers of j would want to defect to other factions despite the reduced payoff with the expanded membership, because otherwise the marginal candidate would have helped someone else. Helpers in other factions would have no incentive to switch allegiance either. Then, $\xi(M)$ can be written as:

$$\xi(M) = \frac{t_M}{\lambda(M-1)}\left(\frac{\sum_{k=1}^{L(M-1)} t_k n_k(M-1) + \sum_{k=L(M-1)+1}^{M-1} t_k}{\sum_{k=1}^{L(M)} t_k n_k(M) + \sum_{k=L(M)+1}^{M} t_k}\right).$$

Because $n_k(M) = n_k(M - 1)$ $\forall$ $k \in [1,L]$ and $k \neq j$, $n_j(M) = n_j(M - 1) + 1$, and $L(M) = L(M - 1)$,

$$\xi(M) = \frac{t_M}{\lambda(M-1)}\left(\frac{\sum_{k=1}^{L(M)} t_k n_k(M) + \sum_{k=L(M)+1}^{M-1} t_k + t_j}{\sum_{k=1}^{L(M)} t_k n_k(M) + \sum_{k=L(M)+1}^{M-1} t_k + t_M}\right)$$

$$= \frac{t_M}{\lambda(M-1)}\left(\frac{\lambda(M)(N-M) + \sum_{k=L+1}^{M-1} t_k + t_j}{\lambda(M)(N-M) + \sum_{k=L+1}^{M-1} t_k + t_M}\right)$$

An increase in t_p increases $\lambda(M-1)$ and therefore reduces the first fraction on the *RHS*. It also increases $\lambda(M)$ and reduces $L(M)$, but since $t_j > t_M$, the ratio in the brackets decreases as well. If, instead of helping a faction leader, the marginal candidate switches to help another singleton candidate, then $t_j = t_{L(M)+1} \geq t_M$, where $L(M)+1$ is the strongest singleton candidate when there are M candidates, with the same result. Therefore, $\xi(M)$ is decreasing in t_p.■

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