# The Shape of Temptation: Implications for the Economic Lives of the Poor*

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## 1 Introduction

The poor often behave as if they are very myopic. For example, consider the stylized fact that they borrow repeatedly (often continuously) at very high interest rates.[1] Under standard consumption theory, borrowing at high rates reveals a preference for consumption today since one could always maintain the same investment level by taking a marginally smaller loan and consuming less. These borrowers must value a dollar of consumption today at least as much as $R$ dollars of consumption tomorrow, where $R$ is the gross rate of borrowing. If individuals are in steady state, the Euler inequality[2]

$$u'(c_t) \geq \delta R u'(c_{t+1})$$

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[1] Aleem (1990) in a survey of money lenders finds an average interest rate of 78.5% per year. Even formal micro-financial institutions charge extremely high rates. In Mexico and other countries, for example, prominent micro-finance institutions charge 90%+ per year. The fact that such borrowing happens repeatedly is seen in the case of agricultural finance (Dreze, Lanjouw and Sharma (1997)), the case of daily working capital (Karlan and Mullainathan (2009)) and payday loan usage in the United States (Skiba and Tobacman (2007)).

[2] Note that the steady state assumption rules out two alternative explanations. First one could assume that steady state borrowers are quickly becoming rich: $U'(c_{t+1})$ is low relative to $U'(c_{t+1})$. Factually speaking this seems reasonable. Another alternative, which is more realistic, is that borrowing is used to deal with shocks, i.e. it occurs only when the need for cash today $U'(c_t)$ is particularly high. While this is surely an accurate feature of some high interest rate borrowing, our focus is on *serial* borrowing, such as for working capital. For example, Karlan and Mullainathan (2009) find street vendors in India borrow every day at rates upwards of 5% day to finance their working capital needs despite having been in business for many years (the median length of time they have been in business is 9 years).

implies that $\delta \leq \frac{1}{R}$. In other words, such a commonplace phenomenon–continuous high interest rate borrowing–directly implies stark myopia ($\delta$ is particularly low).[3] While certainly the easiest to document, high interest rate borrowing is only one of many behaviors that the standard model would interpret as extreme myopia. A second stark example is from the investment domain. A growing literature suggests that the poor fail to self-finance what appear to be high return *divisible* investments or to accumulate enough to buy non-divisible investments. This has been found for working capital amongst micro-entrepreneurs and for fertilizer amongst fertilizers (see de Mel, McKenzie, and Woodruff (2008) on micro-enterprise returns; Lee, Kremer and Robinson (2009) on inventories; Duflo, Kremer and Robinson (2009) on fertilizer; and Udry and Anagol (2006) for a variety of evidence).

In this paper, we argue that a broader framework–not just low $\delta$–is necessary to understand the variety of behaviors. We build on a growing body of research on time inconsistency (Shefrin and Thaler (1981), Laibson (1997), O'Donoghue and Rabin (1999)): individuals are myopic about some decisions and far-sighted about others. The poor are no different in this regard. The same populations that borrow at very high rates also engage in various far-sighted behaviors–from purchase of burial insurance to participation in ROSCAs (Collins et. al. (2009)). Our model focuses on the *interaction* between time inconsistency problems and poverty. Focusing on this interaction requires reformulating existing time inconsistency models of consumption. Most models focus on the level of consumption: today's self is tempted to consume more than the long-run self would want. We focus on the *composition* of consumption: today's self is tempted to consume more today *of certain goods* than the long-run self would want. Certain foods (for example fatty or sugary ones) produce instantaneous, visceral pleasures that create temptations for today's self.[4] We thus assume that consumption $c_t$ has two components; $x_t$ and $z_t$. The first, $x_t$, reflects consumption spending on which there is no temptation: $x_t$ has prospective value for all periods $s \leq t$. The second, $z_t$, reflects consumption where there is temptation; only the $t$ self values $z_t$, i.e. there is no prospective valuation of $z$ consumption.[5]

[3] Complications to the utility function could provide alternative explanations but each appears counter-factual in some way. For example, one could argue that the poor cannot cut back on consumption because they are against some sort of "minimum consumption" constraint. This, however, appears implausible given (a) that consumption actually shows substantial high frequency variation (Collins et. al. (2009)) and (b) the direct evidence (see Banerjee and Duflo (2007)) that even the very poor spend a significant part of their income on what are clearly not survival necessities (cigarettes, alchohol, expensive but not especially nutritious foods). Another argument is a Stone-Geary utility function where the realized discount factor is a consequence of high mortality. This is implausible given the low mortality rates: even for the very poor, the probablity of dying is many orders of magnitude below 5% per day. A very different alternative is to argue that the interest rates are not realized because of high default rates. Factually, default rates for formal and informal institutions are extremely low. This is certainly true of the street vendors in the Karlan-Mullainathan study mentioned above but also the shared experience of MFIs (Morduch et. al. cite).

[4] Note that in our model, we are not focusing on the health consequences of such goods, but merely the fact that these goods cost money.

[5] We show in Section 2.1.1 that this model provides a simple way to generalize the hyperbolic model. Of course, we make no claim that this is the only way to capture the idea that

Consumption in this model is determined by a modified Euler equation:

$$u'(x_t) = \delta R u'(x_{t+1})[1 - z'(c_{t+1})]$$

where $z(c)$ quantifies the amount of total consumption that goes to temptation goods. In other words, spending tomorrow is subject to a "temptation tax": a dollar spent tomorrow is partly dissipated on temptation goods. Since today's self does not value tomorrow's temptation spending, the temptation spending is a waste. High interest rate borrowing could now be due to a low $\delta$ or a high temptation tax $z'(c)$.

The essential contribution of this paper then is to relate the structure of the temptation tax to behaviors. One shape of temptation is particularly interesting, $z'(c)$ decreasing and hence $z(c)$ concave. As individuals consume more, a smaller fraction of total spending will go towards temptation. In particular, the rich may spend more on temptation goods but not proportionally more. One reason for this concavity could be that temptations are primitive consumption urges; fat, sugar, visceral pleasures. Such urges may (after a certain wealth level) be easily satisfied and more expenditures are unlikely to produce much more pleasure. For example, if as a practical matter, donuts can only get so expensive (and even then only gain very little in "temptingness"), the resulting $z(c)$ function would be concave.[6] Relatedly, the "temptingness" of a good may depend on income. Temptations may be those things that are unaffordable for the current intertemporal budget. So as income increases, certain goods can move from being counted as temptation ($z$) consumption to being counted as non-temptation ($x$) consumption. Expenditure on a cup of tea can be a temptation to someone living on a \$1 a day who cannot afford it, but be part of planned (i.e $x$) consumption to someone living on \$10 a day who can. Of course, in the end this assumption must be judged, as is any assumption, by how well it fits the data. We discuss tests of the assumption in greater detail in Section 4.9. For the bulk of the paper we assume $z(c)$ concave.[7] In this paper, we draw out several unique consequences that follow from $z'(c)$ decreasing.

First, the assumption predicts that discount factor estimates according to the traditional Euler equations will be biased in a systematic way: they will appear to correlate more positively with income more than they actually do. For example, even if individuals have the same discount rate and vary only in income, the poor will appear to discount the future more heavily. While this result is a

temptation is not simply about more today versus tomorrow but is embodied in specific goods. Such good-specific temptations could be captured in a hyperbolic model by allowing goods to deliver different types of consumption at different points in time (as in Gruber and Koszegi (2001) on cigarettes). We have taken our approach because it provides a tractable language for thinking about questions such as how the extent of temptation varies with the level of consumption.

[6] In fact expenditure data on oils, fats and sugars in fact suggests that while they are an increasing share of *food* expenditures, they are actually a *decreasing* share of total expenditures (see for example Subramanian and Deaton (2006)).

[7] In Section 3.1 we discuss some interesting commitment consequences of the $z(c)$ linear special case: the demand for commitment devices such as restricted access savings accounts or ROSCAs and an excess demand for durables.

direct consequence of concavity of $z(x)$, it illustrates the analytical rationale for separating $z$ and $x$ goods, rather than working with a quasi-hyperbolic model. We can directly relate an intuitive assumption in the framework—concavity of $z(x)$—to an often observed phenomena—lower discount rates for he poor but with a different interpretation.[8] What appears to be myopic behavior amongst the poor is as much a result of their poverty as it is a cause.

Second, the temptation tax *alone* can generate a behavioral poverty trap. In this case, there will exist a critical wealth level $\underline{w} > 0$ such that long run consumption exhibits a discontinuous jump around initial wealth $\underline{w}$. Those just below $\underline{w}$ will save very little or dissave, while those just above will save a lot more. Note that this poverty trap arises even though we have no lumpy investments or even credit constraints, the usual ingredients of poverty trap models. Individuals face an added incentive to save since if they increase wealth they lower the temptation tax. On the other hand, at low enough wealth, this added incentive is offset by the level of the temptation tax. Put another way, the moderately well-off can save in the hopes of being sufficiently wealthy to avoid a large tax. The poor cannot save enough to accomplish this (or in a multi-period model it would take them too long do so); hence they simply dis-save. People are present-biased because they are poor, but that in turn keeps them poor.

Third, a concave temptation tax affects individuals' responses to uncertainty. Individuals may hold very little buffer stock savings relative to the shocks they face. Future selves will spend liquid savings on temptation goods, which creates a disincentive to hold liquid savings. The model has also surprising implications for precautionary savings behavior: an increase in variance can actually *reduce* precautionary savings. This is true even if the utility function (both $u(x)$ and $v(z)$) exhibit prudence.

Fourth, investment behavior in this framework will be constrained not just by returns and the minimum admissable scale associated with the project, but also the maximum possible scale: a small high return investment may not be as attractive as a larger opportunity with lower returns. This, we feel, can help explain phenomena such as why individuals fail to undertake very high return non-lumpy investments that are available to them (we already mentioned Duflo, Kremer and Robinson's work on fertilizer and Lee, Kremer and Robinson's research on working capital). The model also has implications for debt. While predicting that there will be a desire for commitment (maximum loan sizes) as in any time inconsistency model with sophistication, it also notes this desire is not monotonic: people may prefer future selves to take a bigger loan over a smaller one.

Finally, we illustrate how, in the presence of these preferences, monopolistic money-lenders would have the incentive to prevent the poor from adopting high-return technologies, for the purpose of locking them into a debt trap. As pointed out by Srinivasan (1994), this is not possible with standard preferences.[9]

While these results are qualitative, in the concluding section we discuss

[8] In the economics literature, this has been remarked on as early as Irving Fisher (1932).

[9] The idea of trying to formalize this debt trap goes back to Bhaduri (1977).

potentially quantitative tests of the model. The primary value of this paper is the ability to deliver a number implications from a simple assumption. We feel that the realism of this assumption can also be tested directly. In Section 4.9 we describe in detail two simple experimental tests of our core assumption.

# 2 Model

## 2.1 Modeling Temptations

Before setting up the model in full generality, we describe a simple version that captures the main intuitions. Suppose that an individual lives for two periods $t = 1, 2$. Each period, he can consume $x_t$ and $z_t$, two different components of consumption.[10] If individuals were time consistent and had separable utility functions, they would maximize $u(x^1) + v(z^1) + \delta u(x^2) + \delta v(z^2)$. The two goods produce different utility and the two time periods have different value: today's self favors today over tomorrow. But the two selves agree on how to weight the different goods. In our model, temptation goods are exactly those goods about which today's self and tomorrow's self disagree. A stark way to model this is to assume that the period 1 self maximizes

$$u(x^1) + v(z^1) + \delta u(x^2).$$

In other words, he values his own spending on temptation goods but does not value tomorrow's self's spending on them. One rationale for this is that hedonic experiences can have variable textures. Some components of pleasure are lasting while others provide only immediate gratification; we refer to the latter as temptations. For example, sugary or fatty foods provide an immediate burst of pleasure and may be particularly prone to pleasure. Interestingly, these immediate gratification pleasures appear to be fairly simple things such as taste or sex drives. This interpretation is bolstered by recent neurological research which argues for these varying dimensions of pleasure (see McClure et. al. (2004)). Several authors argue that this neurological evidence favors a two-self model in which one self is tempted by visceral pleasures and the other is focused on longer term consumption (Fudenberg and Levine (2006)). We provide a particular way to model this disjunction.

There are other ways to model this idea. For example, one could micro-found this within the context of a hyperbolic model where some goods provide more utils today and others provide fewer. We chose this particular framework because it has three advantages. It allows us to introduce between-good variation in the extent of the self-control problem associated with the goods. Second, it allows us to capture the idea that self-control problems vary by the level of consumption. Third, it allows us to introduce self-control problems in a two period model, unlike in the standard hyperbolic discounting model (where individuals maximize a utility function of the form $u(c_0) + \beta \sum_t \delta^t u(c_t)$). We do not

[10] Note in what follows these two components will be two different amounts of money spent on consumption.

claim that these ideas cannot be captured in other, more standard models of time inconsistency. As we discuss in section 3.1, the idea that temptations vary with the level of consumption, for example, can be captured in the hyperbolic discounting framework; the conditions required to get the property are just less transparent than in ours.

The more general model simply recognizes that when there are several goods, one can form an index of spending on temptation goods and an index of spending on non-temptation goods.[11] To capture these ideas, suppose there are n goods consumed in each of two time periods.[12] Denote the amount consumed of good $i = \{1, 2.....m, m+1, ....n\}$ in time $t = \{1, 2\}$ by $x_i^t$. At time 2 individuals maximize:

$$E\{\widetilde{U}(x_1^2, ..., x_m^2)\} \tag{1}$$

where the $W$ is increasing and concave, and $E\{\cdot\}$ is the expectations operator defined in terms of the uncertainty in the model, as defined below.

At time 1, we assume individuals have decision utility equal to:

$$\widetilde{U}(x_1^1, ..., x_m^1) + \widetilde{V}(x_{m+1}^1, ..., x_n^1) + \delta E\{\widetilde{U}(x_1^2, ..., x_m^2)\} \tag{2}$$

To understand this utility, it is useful to contrast equation (2) with what would happen in a traditional discounted utility model:

$$\widetilde{U}(x_1^1, ..., x_m^1) + \widetilde{V}(x_{m+1}^1, ..., x_n^1) + \delta E\{\widetilde{U}(x_1^2, ..., x_m^2) + \widetilde{V}(x_{m+1}^2, ..., x_n^2)\}$$

The key difference with our formulation is that the term $E\{\widetilde{V}(x_{m+1}^2, ..., x_n^2)\}$ is omitted. Thus for goods $i = 1, .., m$, the time 1 self uses the same utility function as the time 2 self and discounts the future by the same rate, $\delta$. However, the time 1 self places *no* weight on the utility derived from goods $i = m+1, ..., n$ in period 2. Thus, goods $m+1, ..., n$, are the goods which the future self values but which today's self assigns zero weight.[13] We will refer to these goods as "temptation goods".

Considerable simplification is possible if we make use of within period optimization. Choosing units so that all good prices are 1, we define the indirect utility functions

$$U(x^t) = \max_{x_1 \ldots x_m} \widetilde{U}(x_1^1, ..., x_m^1), \sum_{i=1}^{m} x_i^t = x^t, t = 1, 2$$

and

$$V(z^t) = \widetilde{V}(x_{m+1}^2, ..., x_n^2) \max_{x_{m+1} \ldots x_n}, \sum_{i=m+1}^{n} x_i^t = z^t,$$

[11] In Section 2.6 we consider a model where goods have both temptation and non-temptation components and in this case the indices are merely a measure of the dollars spent on temptations.

[12] This setup can be obviously extended to more periods.

[13] A more general formulation would endow goods with a variable weight but for our purposes this simple model where there are goods with full weight and goods with no weight is particularly convenient.

The key insight is that the total amount spent on any subset of goods must be spent optimally within that subset, even when the consumer is actually deciding over a larger set. We can therefore write the above maximand in the compact form

$$U(x^1) + V(z^1) + \delta E\{U(x^2)\}. \tag{3}$$

For most of this paper, this is the utility function we will use. In essence, $x$ denotes the index of spending on non-temptation goods, while $z$ denotes the index of spending on temptation goods, much in the same way that $c$ denotes the spending on total consumption in traditional and hyperbolic discounting models.[14] That both $U$ and $V$ should be increasing and concave follows from the corresponding assumptions about the $W_i$. Futhermore we will assume in everything we do that $U$ is at least three times differentiable everywhere. We refrain from making the corresponding assumption for the $V$ function to accomodate certain special cases.

### 2.1.1 Relation to hyperbolic discounting

As noted earlier, this model shares many features with a hyperbolic discounting model. In this particular context, however, these similarities are hard to make precise since in two periods, the hyerbolic discounting model is indistinguishable from an exponential discounting model. Time inconsistency in hyperbolic discounting models comes from the fact that "next period's self" puts too little weight on the subsequent "self" from the point of view of today's self — therefore when there is just the next period's self, nothing interesting happens.

To see the similarities, therefore, consider a T-period extension of our model. Under assumptions that gave us (3), the intertemporal maximand for this case can be written in the form

$$U(x^1) + V(z^1) + \sum_{t=1}^{T} \delta^t E\{U(x^t)\}$$

Maximizing $U(x) + V(z)$ subject to a budget constraint $x + z = c$, and the conditions $(x \geq 0, z \geq 0)$ gives us a function $x(c)$.

$$\begin{aligned} \widetilde{U}(c) &= U(x(c)) \\ & \quad and \\ \widetilde{V}(c) &= V(z(c)) \end{aligned}$$

Then the above expression becomes

$$\widetilde{U}(c^1) + \widetilde{V}(c^1) + \sum_{t=2}^{T} \delta^t E\{\widetilde{U}(c^t)\}$$

[14] As a result, there are several underlying good-specific utility functions that can generate the index utility described in (3). Thus while the separable utility formulation in (2) generates it, other more complicated utility functions can also give rise to it.

When $\widetilde{V}(c) = \alpha \widetilde{U}(c)$, this is the standard $\beta$-$\delta$ utlity function with $\beta = \frac{1}{1+\alpha}$. One case where this condition holds is when

$$\begin{aligned} U(x_t) &= \frac{x_t^{1-\alpha}}{1-\alpha} \text{ and} \\ V(z_t) &= A\frac{z_t^{1-\alpha}}{1-\alpha}. \end{aligned}$$

In other words two essentially identical CRRA functions: Under these assumptions, within period choice between $x$ and $z$ will give us $z_t = qx_t$ where $q = A^{\frac{1}{\alpha}}$. Substituing this into our maximand gives us

$$\begin{aligned} & \frac{x_0^{1-\alpha}}{1-\alpha} + \frac{(q)^{1-\alpha} x_0^{1-\alpha}}{1-\alpha} + \sum \delta^t \frac{x_t^{1-\alpha}}{1-\alpha} \\ = & \ (1+q^{1-\alpha})[\frac{x_0^{1-\alpha}}{1-\alpha} + \sum \beta\delta^t \frac{x_t^{1-\alpha}}{1-\alpha}] \end{aligned}$$

where $\beta = \frac{1}{1+q^{1-\alpha}}$, which is exactly in the hyperbolic form.

More generally, the models will be distinct, since they represent different assumptions about temptation. The hyperbolic model focuses on disagreements about the *level* of consumption whereas our model focuses on disagreements about the *composition* of consumption. There are two ways to understand this difference. First, we could start by assuming that our model is the more accurate model of temptation–that temptation in fact occurs at the level of individual goods. In this case, the hyperbolic model should be seen as a convenient approximation to the T-period version of our model and $\beta$ in those models is an approximation to a more endogenously determined myopia factor (such as the temptation tax in our model). Alternately we could start from the premise that the hyperbolic model is the right framework but in *util* space: individuals value more utils today than the long-run self would want them to. Different goods provide different time-profiles of hedonic flows. Hence some goods are more tempting because they provide more present utils relative to the long-run utils they provide. Our model then is a reduced form that abstracts from the differential time-flow of utils. In this paper, we do not attempt to distinguish between these alternative positions and simply work with our $U(x)$ and $V(z)$ framework for the rest of the paper.

## 2.2 Maximization

Individuals maximize this utility subject to their budget constraint. In the first period they earn a deterministic "labor" income $y^1$. In that period they also have the opportunity to take a loan, which they can use to make an investment or simply consume more than they currently have. Let the amount of investment be $k^1$, and let $w^1 = y^1 - x^1 - z^1$ be his savings. Now if $w^1 \geq k^1$ then he is a net lender to the market. Otherwise, he is a net borrower. Now define the credit supply function as follows: Let $r(w^1, k^1)$ be the interest rate paid per unit of net borrowing from the market $(k^1 - w^1)$. We assume that this function is defined both for positive and negative values of $k^1 - w^1$ : when $k^1 - w^1$ is negative, this is interest he earns on his lending to the market. Also let $F^1(k^1, \theta)$ represent the intertemporal "production" technology that the person has access to which is a function of the total amount invested ($k^1$) and some potential shock ($\theta$) that we assume is realized before investment decisions are made but after savings is chosen. We impose the assumption that $F(k^1, \theta)$ is differentiable, increasing and (weakly) concave in $k^1$ and that $r(k^1, w^1)$ is differentiable, increasing and (weakly) convex in $k^1$ and differentiable, decreasing and concave in $w^1$ to rule out poverty traps that result from non-convexities in production (as in Galor-Zeira (1993)) or in the credit supply function (as in Banerjee-Newman (1994)).

Now we can define a resource generation function $f(w^1, \theta)$:

$$f(w^1, \theta) = \max_{k^1 \geq 0} \{F(k^1, \theta) - r(w^1, k^1)(k^1 - w^1)\} \tag{4}$$

The assumptions made above together imply that $f(w^1, \theta)$ is differentiable, increasing and (weakly) concave in $w^1$.

Note that this definition encompasses a number of very different cases. One is the case of perfect capital markets–$r(w^1, k^1)$ is a constant. A second is where there is only borrowing for consumption. This is the scenario where $F(k^1, \theta) \equiv 0$. In this case, $k^1 = 0$ is clearly optimal and therefore all borrowing is for consumption. Finally, we can choose the $r(w^1, k^1)$ function to approximate a curve that becomes vertical at some fixed value of $k^1$ for each value of $w^1$. This is the case of a credit limit.

In the second period, the person also gets a potentially uncertain "labor" income $y^2(\theta')$. We assume that $\theta$ and $\theta'$ are independent random variables and that $\theta'$ is realized in the second period before consumption decisions are taken.

As defined above, maximizing $U(x^2) + V(z^2)$ subject to a budget constraint $x^2 + z^2 = c^2$, and the conditions $(x^2 \geq 0, z^2 \geq 0)$ gives us functions $x^2(c^2)$ and $z^2(c^2)$. Under the standard assumption that both $U$ and $V$ are strictly concave, $x^2(c^2)$ and $z^2(c^2)$ will be non-decreasing in $c^2$. If $V$ is also differentiable (in addition to $U$ being differentiable), then $x^2(c^2)$ and $z^2(c^2)$ will be differentiable and strictly increasing everywhere except perhaps where the non-negativity constraint binds. Using the fact that $c^2 = f(w^1, \theta) + y^2(\theta')$, we can write this as $x^2(f(w^1, \theta) + y^2(\theta'))$. Also for future use, define $z^2(x^2)$ to be the function that is defined by the first order condition for maximizing $U(x^2) + V(z^2)$ subject to a budget constraint $x^2 + z^2 = c^2$, i.e by the equation $V'(z^2) = U'(x^2)$, and define $W(c)$ to be the indirect utility function defined by maximizing $U(x) + V(z)$ subject to a budget constraint $x + z = c$, and the conditions $(x \geq 0, z \geq 0)$. Since both $U$ and $V$ are increasing and strictly concave, so is $W(c)$.

The decision-maker in the first period is assumed to be sophisticated and therefore takes this function into account in making his first period choices. We assume that in the first period he gets an income/endowment $y^1$.

Therefore in the first period, the decision problem is to maximize

$$U(x^1) + V(z^1) + \delta E_{\theta, \theta'}\{U(x^2(f(w^1, \theta) + y^2(\theta')))\}$$

subject to

$$w^1 = y^1 - x^1 - z^1$$

and

$$x^1 \geq 0, z^1 \geq 0.$$

## 2.3 First order conditions

If an interior optimum exists and $\frac{dx^2(f(w^1,\theta))}{df(w^1,\theta)}$ and $\frac{dV(z^1)}{dz^1}$ exist at the optimum, then the folloiwng conditons must hold.

$$\begin{aligned}
\lambda &= \frac{dU(x^1)}{dx^1} \\
\lambda &= \frac{dV(z^1)}{dz^1} \\
\lambda &= \delta E_{\theta,\theta'}\{\frac{dU(x^2(f(w^1,\theta)+y^2(\theta')))}{dx^2} \\
&\quad \times \frac{dx^2(f(w^1,\theta)+y^2(\theta'))}{df(w^1,\theta)}\frac{df(w^1,\theta)}{dw^1}\} \\
w^2 &= f(w^1,\theta)+y^1-x^1-z^1
\end{aligned}$$

When the differentiability condition fails something similar holds with appropriately defined left-hand and right-hand derivatives.

These conditions can be rewritten in the more compact form:

$$\begin{aligned}
\frac{dU(x^1)}{dx^1} &= \frac{dV(z^1)}{dz^1} \\
\delta E_{\theta,\theta'}\{\frac{dU(x^2(c^2))}{dx^2}\frac{df(w^1,\theta)}{dw^1}\frac{dx^2(c^2)}{dc^2}\} &= \frac{dU(x^1)}{dx^1} \qquad (5)
\end{aligned}$$

## 2.4 The Modified Euler Equation

The condition

$$\delta E_{\theta,\theta'}\{\frac{dU(x^2(c^2(\theta,\theta'))}{dx^2}\frac{df(w^1,\theta)}{dw^1}\frac{dx^2(c^2(\theta,\theta'))}{dc^2}\} = \frac{dU(x^1)}{dx^1} \qquad (6)$$

where $c^2(\theta,\theta') = f(w^1,\theta) + y^2(\theta')$ ought to be reminiscent of the standard Euler equation in dynamic consumer maximization problems. Indeed, the only difference comes from the presence of the term $\frac{dx^2(c^2)}{dc^2}$ : In our setting the standard Euler equation would take the form

$$\delta E_{\theta,\theta'}\{\frac{dU(x^2(c^2(\theta,\theta')))}{dx^2}\frac{df^2(w^1,\theta)}{dw^1}\} = \frac{dU(x^1)}{dx^1}.$$

The difference comes from the fact that there is some "dropped utility"—only part of the total expenditure on period 2 goods is valued by the period 1 self. Since $\frac{dU(x^2(f(w^1,\theta)))}{dx^2}$ and $\frac{df(w^1,\theta)}{dw^1}$ are always non-negative and $\frac{dx^2(c^2)}{dc^2} \leq 1$, this has the immediate implication that an observer who uses the modified Euler equation to estimate the decision-maker's discount factor as if it was the standard Euler equation (i.e proxying it by the ratio $\widehat{\delta} = \frac{\frac{dU(x^1)}{dx^1}}{E_\theta\{\frac{dU(x^2(c^2(\theta,\theta')))}{dx^2}\frac{df(w^1,\theta)}{dw^1}\}}$), would think that the person is more impatient than he actually is ($\widehat{\delta} \leq \delta$). Moreover, it does not matter whether he uses an $x$ or a $z$ good to estimate the discount factor since, from the within period maximization $\frac{dU(x^2(c^2(\theta,\theta')))}{dx^2} = \frac{dV(z^2(f(w^1,\theta)))}{dz^2}$ and $\frac{dU(x^1)}{dx^1} = \frac{dV(x^2)}{dx^2}$

In essence, the tempted consumer faces a temptation tax: he knows that future resources will be wasted on consumption that he does not care about. Sophistication about his temptations leads him to incorporate this tax on spending now. Note that naivete about these temptations could produce different results. We conjecture that naive consumers would reproduce several of the results below but through a different mechanism. The sophisticated consumer fails to save because he foresees the tax; the naive consumer would save but the savings would then be "unexpectedly" wasted on temptations. In both cases, savings towards long term goals is thwarted. Examining this conjecture about naive consumers analytically is left for future work.

## 2.5 The strength of temptations: A revealing special case

Another immediate payoff from having derived the Modified Euler Equation is that it allows us to introduce the idea of being tempted to a greater or lesser extent. Assume that $U$ and $V$ are both CRRA with the same coefficients:

$$\begin{aligned} U(x) &= \frac{x^{1-\alpha}}{1-\alpha} \text{ and} \\ V(z) &= A\frac{z^{1-\alpha}}{1-\alpha}. \end{aligned}$$

We already observed that in this case $\frac{dx^2(c^2)}{dc^2}$ is a constant, and the preferences (in the T-period case) have exactly the hyperbolic discounting form. Assume that we are in this case and therefore $x(c) = \frac{1}{1+q}c$ and $z(c) = \frac{q}{1+q}c$ where $q = A^{\frac{1}{\alpha}}$. Moreover let the person have no prospect for production ($F(k^1,\theta) = 0$) and no second period income. His only means of transferring wealth to the second period is by saving in first period at the given interest rate $R$. Hence (using the fact that the person consumes everything he has in period 2 and therefore $c^2 = Rw^1$), the Modified Euler Equation can be rewritten in the form

$$\delta R\frac{1}{1+q}(\frac{Rw^1}{1+q})^{-\alpha} = (x^1)^{-\alpha} \tag{7}$$

In addition to this condition we have the inter-temporal budget constraint, which is

$$y^1 - (1+q)x^1 = w^1.$$

What is the effect of being more tempted on savings in this environment? In part that will depend on how we measure temptation. In the current CRRA model $\frac{q}{1+q}$ would appear to be a natural measure, since it represents the share of second period consumption that is wasted from period 1's point of view. The problem however is that changing $q$ also changes the within period utility function in both periods and this might have implications for savings even in the absence of self-control problem. Conveniently, It turns out that in the CRRA case, a change in $q$ shifts the period by period indirect utility functions in the same proportion (by a factor $(1+q)^{\alpha}$) in all periods and therefore has no effect on any of the intertemporal choices through this route. Therefore in this case we can focus on the effect of changing $q$.

Notice that $1+q$ enters (7)in 3 separate places: twice in the term $\delta R\frac{1}{1+q}(\frac{Rw^1}{1+q})^{-\alpha}$ and once through the expression $(1+q)x^1$. The effect through $\delta R\frac{1}{1+q}(\frac{Rw^1}{1+q})^{-\alpha}$ is the effect of an increase in temptation in the second period: Here, there are two pieces—the $\frac{1}{1+q}$ term captures the substitution effect of the"tax" on every dollar that is spent in the second period (the part that goes in to $z$ consumption). The $(\frac{Rw^1}{1+q})^{-\alpha}$ term captures the income effect of the "tax"— less $x$ is consumed because there is more going to $z$. The reader will notice that these two effects have an exact parallel in the effects of a tax on interest earnings. Whether the net effect is to discourage savings or to encourage them, depends on whether or not $\alpha \leq 1$. When the inequality holds, savings will go down with more temptation as a result of this effect. In other words, simply increasing the second period level of temptation keeping the first period level of temptation fixed might actually encourage savings through the "income" effect: people save more because they know that they will waste more and therefore less will end up in the use that the patient self favors.

There is however a third effect: This is the effect through the $x^1(1+q)$ term, which is the effect of an increase in first period $q$. It captures the fact that a higher $q$ in the first period means that keeping $c^1$ fixed, first period spending on $x^1$ has to go down. This is another "income" effect–resulting now from an increased $q$ in the first period–and clearly pushes for a higher level of first period spending (and hence less savings), for any fixed value of second period $q$.

When $q$ changes in both periods the two income effects exactly cancel out in this CRRA case. This is easily seen by rewriting the first equation 7 in the form $\delta(\frac{R}{1+q})(\frac{c^2}{1+q})^{-\alpha} = (\frac{c^1}{1+q})^{-\alpha}$, using the fact that $c^1 = x^1(1+q)$ and then noting that $(1+q)^{-\alpha}$ terms cancel from both sides to leave us with

$$\begin{aligned}
\delta(\frac{R}{1+q})(c^2)^{-\alpha} &= (c^1)^{-\alpha} \\
y^1 - c^1 &= w^1
\end{aligned}$$

We are now left with only one effect of raising $q$ which is the substitution effect of the tax and this always leads to reduced savings: The net effect of increased temptation in this CRRA model is therefore exactly parallel to the effect of greater impatience (i.e lower $\delta$).

The simplicity of this result owes a lot to the CRRA formulation. In the more general case the equivalent of raising $q$ will be to move the entire $z(c)$ function up and it is not clear whether it is possible to do so without introducing shifts in the within period utility functions that would change savings behavior even if there were no dropped utility.

For this reason, we will avoid making this kind of comparison in the rest of this paper: we will limit ourselves to showing that the presence of dropped utility introduces a set of possibilities that could not arise in its absence. However, we go through a small detour before we come to that. The next sub-section argues that the very rigid distinction between temptation ($z$) goods and other ($x$) goods is not necessary for what follows.

## 2.6 Generalizing the idea of the temptation good

The Modified Euler Equation makes it easy to see exactly how we can generalize the preferences assumed above. In particular, we have so far imposed the assumption that there is an entirely separate category of goods that we call temptation goods. In fact our framework can accomodate the possiblity that circumstances determine whether and to what extent, a particular good constitutes a temptation.

To get at this idea, denote by $u^F(x_1, ..., .x_n)$ the utility function that represents the preferences of the forward-looking self over any future outcomes, and by $u^T(x_1, ..., x_n)$ the utility of the current self over current outcomes; both utility functions are now defined over the same set of goods. $u^T$ is where the temptations come in (hence the superscript).

Let $x^T(c)$ be the vector of goods that maximizes $u^T(x_1, ..., x_n)$ subject to $\sum_{i=1}^{n} x_i = c$. Define $w^T(c)$ to be the indirect utility of the tempted utility function: $w^T(c) = u^T(x^T(c))$. Given this definition it is easy to write the maximization problem of the forward-looking decision-maker as one of choosing $(c^1, c^2)$ to maximize

$$u^T(x^T(c^1)) + \delta u^F(x^T(c^2))$$

subject to the intertemporal budget constraint. It is clear that the slope of the $u^P$ function relative to the slope of the of $u^T$ function plays a role similar to the discount factor. We impose the assumption that this does not cause any bias towards the future:

$$\sum_i \frac{\partial u^T(x_i^T(c))}{\partial x_i} \frac{\partial x_i^T(c)}{\partial c} \geq \sum_i \frac{\partial u^P(x_i^T(c))}{\partial x_i} \frac{\partial x_i^T(c)}{\partial c}, \text{ for all } c.$$

Note that both sides of this inequality are evaluated at the same value of $c$.

Assume for the sake of the exposition that the decision maker has a fixed endowment $y$ that he gets in the first period, no second period earnings, and can lend at the rate $r$. Then the first order condition for this maximization problem will be

$$\frac{dw^T(c^1)}{dc} = \delta r \sum_i \frac{\partial u^P(x_i^T(c^2))}{\partial x_i} \frac{\partial x_i^T(c^2)}{\partial c}.$$

It turns out that this decision maker would behave exactly like a $U$-$V$ decision maker with appropriately chosen $U$ and $V$ functions. To see this, note that this would require that the $U(x)$ function and the $x(c)$ function satisfy

$$U'(x(c^2))x'(c^2) = \sum_i \frac{\partial u^P(x_i^T(c^2))}{\partial x_i} \frac{\partial x_i^T(c^2)}{\partial c}$$

and

$$U'(x(c^1)) = \frac{\partial w^T(c^1)}{\partial c}$$

Putting these together gives us that $x(c)$ needs to satisfy the differential equation:

$$x'(c) = \frac{\sum_i \frac{\partial u^P(x_i^T(c))}{\partial x_i} \frac{\partial x_i^T(c)}{\partial c}}{\frac{\partial w^T(c)}{\partial c}}$$

which pins down $x(c)$ if we impose the boundary condition $x(0) = 0$. $x(c)$ is clearly an increasing function and therefore has an inverse function $c^{-1}(x)$. Moreover since

$$\frac{\partial w^T(c)}{\partial c} = \sum_i \frac{\partial u^T(x_i^T(c))}{\partial x_i} \frac{\partial x_i^T(c)}{\partial c} \geq \sum_i \frac{\partial u^P(x_i^T(c))}{\partial x_i} \frac{\partial x_i^T(c)}{\partial c}$$

by our above assumption, $x'(c) \leq 1$ for all $c$.

And given $c^{-1}(x)$, $U(x)$ can be defined according to:

$$U'(x) = \frac{\partial w^T}{\partial c}(c^{-1}(x))$$

along with the boundary condition

$$U(0) = w^T(0).$$

It is true that we have not yet defined $V(z)$ but this is not an issue because Proposition 2.6 below tells us that as long as $x(c)$ is increasing and less than or equal to $c$ for all values of $c$ (both properties that have already been verified above for the $x(c)$ function we constructed),[15] there is always an increasing and concave $V(z)$ function, such that maximizing $U(x) + V(z)$ subject to $x + z = c$ will give us that particular $x(c)$ function.

To summarize, the model where both selves care about all the goods (but not to the same extent) is observationally equivalent to a model with $U$-$V$ preferences (as long as we only observe total expenditures in each period, as

[15] $x(c) \leq c$ follows from the fact that $\frac{dx(c)}{dc} \leq 1$ and $x(0) = 0$.

opposed to the amount spent on each good), for an appropriate choice of $U$ and $V$. This ought to be intuitive: essentially, $x$ captures the part of the expenditure that aligns with what the forward-looking self wants, while $z$ is the part that is wasted from the forward-looking self's point of view, and it should not matter that the wasted expenditure takes the form of excess spending on goods that the forward-looking self also values (just not enough to justify that level of spending).

An implication of this reformulation is that the same good may end up being a temptation good in some contexts but not in others. For example, it is not implausible that for some poor people, nutritious food is a necessity and fancy exercise machines are temptation goods, while the reverse is true of some wealthy and overweight people.

# 3 The Shape of Temptation

The logic so far highlights the effect of a temptation tax imposed by future selves. Though frameworks may differ, this logic is common to most models of self-control: a dissonance between how the time $t$ self would like $t+1$ to spend resources and how the $t+1$ self actually spends money. This framework allows us to talk about the "shape" of temptation.

The shape of the temptation is captured by the shape of the $z(c)$ (or $x(c)$ function). The next result shows that the $z(c)$ function can in principle take any shape that we happen to pick for it as long as it is increasing, non-negative and $z(c) \leq c$. Moreover this remains true even if we fix the shape of the $U(x)$ function and only vary the shape of the $V(z)$ function within the class of increasing, concave functions.

**Proposition 1** *Assume that the $U$ function is known and fixed. Let $z(c)$ and $x(c)$ be a pair of non-negative valued, strictly increasing functions defined on $c \in [0, C]$ for some $C > 0$, such that $z(c) + x(c) = c$. Then there exists an increasing, differentiable and strictly concave function $V$ defined on $[0, z(c)]$ such that the assumed $z(c)$ and $x(c)$ functions are the result of maximizing $U(x) + V(z)$ subject to a budget constraint $x + z = c$, and the conditions ($x \geq 0, z \geq 0$)*

***Proof.*** *Define the function $g(z) = x(h(z))$ where the function $h(z)$ is the inverse of the function $z(c)$,which exists because of the strict monotonicity of $z$. Then define*

$$V(z) = \int_0^z U'(g(y))dy$$

*Clearly $V'(z) = U'(g(z)) > 0$. It is concave because when $z$ increases $g(z)$ increases and $U'(g(z))$ decreases.* ■

Note that we did not require that $z(c)$ be differentiable, and for that reason, $V$ may not be twice differentiable.

With this result in hand, the rest of the paper studies the implication of different shapes for the $z(c)$ function in the model of consumption and savings introduced above. The key distinction, it turns out, is between $z'(c)$ decreasing and $z'(c)$ constant, though in the analytic results below we distinguish between $z'(c)$ decreasing and $z'(c)$ non-decreasing. Constant $z'(c)$ means that irrespective of income individuals face the same tax. This would mean that the availability (and utility) of temptation goods scale linearly with the consumption bundle. Decreasing $z'(c)$ on the other hand means that as individuals consume more, temptations are a smaller fraction of consumption: $\frac{z(c)}{c}$ is declining with income.

## 3.1 Constant Temptations

Though our primary results come from declining temptations, the constant temptations case is also interesting if somewhat less novel. Many of what follows has already been discussed in the literature or is a ready implication of existing frameworks. It is reassuring, however, that these results follow easily from our framework. Moreover, they serve as a useful contrast to the results that require declining temptations. The core implication of the constant temptation case is the demand for commitment, though this demand can take several forms.

First, the constant temptation assumption points out individuals' willingness to pay for illiquid durables. To see this, let's return to our original framework where there were multiple $x$ goods, each with an associated separable utility function. Assume now that there two $x$ goods, $x_1$, which is non-durable and $x_2$, wchich represents a durable constant returns consumption technology in which 1 unit of investment at time 1 pays $u_d$ units of additively separable $x$ utility in both periods 1 and 2. In this case an indvidual would be willing to pay up to $\frac{u_d}{u'(x_1^1)}(1+\delta)$ for this one unit. Because it is committed $x$ consumption, it is discounted at rate $\delta$. Interestingly, this would contrast with the discount factor implied by the modified Euler equation $\delta(1 - z'(c_t))$. In other words, people appear to be more willing to invest in consumption durables than in a generic income generating technology. This, we feel, is an important fact to keep in mind as it suggests inconsistencies in which people might be unwilling to take up high return income investments but be willing to save in order to purchase consumption durables. The logic is simple: income is taxed by temptation, consumer durables provide an implicit commitment to $x$ consumption.[16]

Second, empirical evidence suggests that individuals may be interested explicitly in commitment devices. Ashraf, Karlan, and Yin (2004) provide a beautiful illustration of this. They show that individuals who already have bank accounts take-up and utilize a second "SEED" account whose primary advantage is illiquidity: individuals cannot withdraw deposits at will but can only do so when a (personally set) predetermined date or target amount is reached. Demand for commitment savings account can be a consequence of constant temptation. The exact structure of this account, however, requires an analyti-

[16] Note that this relies on our assumption that consumer durables provide additively separable utility. To the extent that durables provide utility that can be substituted for by future consumption choices, they would provide less commitment value and be demanded less.

cal step. In a simple constant temptations model with only concave production technologies, individuals would demand a very specific form of commitment. If they have income or wealth in large amounts, they would value a commitment device which pays out in a steady flow so as to smooth consumption to future periods. In other words, they would want a savings account which transforms a large lumpsum into an annuity like payment. To generate demand for the SEED accounts in Ashraf, Karlan and Yin requires further assumptions. Those accounts allow arbitrary deposits but only allow withdrawals at a specific date or specific amount.

To make sense of the date-based commitment accounts, one would need to assume that there are times when the value of non-temptation consumption is particularly large ($\frac{U_t'(x)}{V_t'(z)}$). Interpreting their results in this light provides a useful insight into the nature of temptation. 70% of individuals in their model choose date-based goals and report uses such as Christmas, Birthday, Celebration or Graduation. Interpreted through our model, this suggests that festival and "party" spending broadly are actually *non-temptation* spending. Of course, such expenditures may represent inefficiencies if there are social externalities, but from the individual perspective it appears that this spending represents the kind of spending individuals would like more, not less, of. To make sense of the amount-based goals, one would need to further assume some form of increasing returns, e.g. a lumpy consumer durable.[17] Absent such an assumption, it is unclear why individuals with constant temptation would value transforming small amounts of cash into a large amount.

Finally and related to amount-based goals, ROSCAs could be interpreted as a demand for commitment (Gugerty (2007) and Basu (2008)). This interpretation makes sense in a constant commitment world, however, only in the presence of some increasing return technology (for example a lumpy consumer durables). Absent this, (constantly) tempted individuals would not value transforming small amounts into one large amount.

## 3.2 Declining Temptations

While these results are interesting, the crux of our framework is the declining temptation case. The key advantage of our framework is to be able to intuitively model how the temptation "tax" changes as income rises. In the Modified Euler Equation (6), the tax is embodied by $\frac{dz^2(c^2)}{dc^2} = 1 - \frac{dx^2(c^2)}{dc^2}$ : for every dollar spent $\frac{dz^2(c^2)}{dc^2}$ is the tax imposed by the period 2 self. Whether the tax declines with overall consumption therefore depends on whether $\frac{dz^2(c^2)}{dc^2}$ decreases or increases with income. Specifically if $\frac{dz^2(c^2)}{dc^2}$ decreases with total consumption, that is to say if $z^2(c^2)$ is concave, the impact of the tax decreases as consumption rises. Put differently, this offers an intuitive way to model the idea that as overall income rises, self control problems lessen. In the rest of the paper we

[17] Such durables would need to demonstrate increasing returns, unlike the durables just discussed which had constant returns.

will contrast this case which we will call the diminishing temptation case (DTC) with the alternative case where temptation does not decline ($\frac{dz^2(c^2)}{dc^2}$ is constant or increasing with $c^2$) which we will call the non-diminishng temptation case (NDTC). Note that NDTC includes the case where $z^2(c^2) = 0$, i.e there are no temptations. Obviously there are many other cases where $z^2$ is neither convex nor concave everywhere which fall into neither of these categories.

What does it mean for $z^2(c^2)$ to be concave? The following result offers an alternative and perhaps more intuitive characterization:

**Proposition 2** *Assuming that $V$ and $U$ are three times differentiable everywhere and $z^2(c^2)$ is twice differentiable everywhere. Then $z^2(c^2)$ is strictly (weakly) concave everywhere if and only if $\widetilde{V}(x^2) = V(z^2(x^2))$ is a strict (weak) concave transform of $U(x^2)$.*
***Proof.*** *In appendix.* ■

In other words our condition is equivalent to assuming that $V$ is more concave than $U$ in this specific sense. So, for example, if both of them are CRRA, then we are asking the coefficient of relative "risk-aversion" on $V$ to be greater than that on $U$.

Why should we believe that $z^2(c^2)$ is concave or equivalently $V$ is more concave than $U$? One argument is based on the idea that most temptations are essentially visceral, reflecting desires that are rooted in our physiology (things like the the desire for sex, the craving for sweets and the love of fatty foods), and for that reason, relatively insensitive to the variety and range of quality that modern market economies offer (this should be true, for example, if the relevant physiological structures evolved in a world where the set of consumption choices was quite limited). As a result it is hard to spend a lot of money on temptation goods without hitting satiation. Another argument is based on the idea that most really expensive goods, like a sports car or a house, are not really available for an impulse purchase in the same sense in which a cup of sugary tea or a trinket is—there are always multiple options that need to be examined and weighed, and all of that ensures that there is time for reflection and reconsideration.

In any case, whether this assumption holds is an empirical question. One tantalizing piece of evidence is provided by data on expenditure shares. For example, Deaton and Subramanian (1996) provide an early analysis of food expenditures. They show that in rural Maharashtra, the poorest decile of rural households spends 12.2% of their total expenditures on sugar, oils and fats. These are goods that could plausibly be thought to have *some* temptation component[18] (or at very the least to have more temptation value than cereals or pulses). For the richest decile, who are also by no means rich, even by Indian standards, this number is 8.7%. While there surely are other temptation goods,

[18] In Section 4.9 we return to the central issue of how one might be able to determine which goods are temptation goods.

this declining fraction illustrates our core assumption.[19] Another interesting category is leisure, which is plausibly, at least in part, a $z$ good: There is some evidence that leisure hours decrease with income (Banerjee and Duflo (2007)). For example, in rural Indonesia, those living at less than a \$1 a day work 34.6 hours a week while those living on 2 to 4 dollars a day work 40.8 hours.[20]

There are two other ways to test the framework. First, in the next section we present a set of direct and testable consequences of $z^2(c^2)$ being concave. Testing them is a way to jointly test our model and this assumption. A second approach, which we take up in Section 4.9 is more direct. We describe how, using household consumption data and a set of choice experiments, one could plausibly determine the set of goods that are relatively more tempting. Examining whether these goods are a decreasing share of the budgets of the rich allows us to directly test the assumption that $z^2(c^2)$ is concave.

### 3.3 Back to the comparison with the hyperbolic model

To end this section we return to the comparison with the hyperbolic model: We ask what would it take to have a declining "tax" on future marginal utility in the presence of hyperbolic discounting. To do this we need a model with three periods, since the hyperbolic part only kicks in when there are more than two periods. Therefore let the decision maker maximize

$$U(c_1) + \beta\delta U(c_2) + \beta\delta^2 U(c_3)$$

subject to the budget constraint

$$c_1 + c_2 + c_3 = w.^{21}$$

As is well-known (from Harris-Laibson (2001) for example), the optimal consumption path for this decision-maker is characterized by the following first order condition

$$U'(c_1) = \beta\delta U'(c_2)[1 - \beta c_2'(w_2)]$$

where $c_2(w_2)$ is defined by the second period decision maker's first order condition

$$U'(c_2(w_2)) = \beta\delta U'(w_2 - c_2)$$

and $w_2 = w - c_1$.

[19] If temptations were primarily about food, this would suggest a different assumption on $z(c)$. At extreme levels of poverty, marginal income is surely spent on necessary calories. In this case $z(c)$ would no longer be concave but $S$-shaped. At very low levels of income, $z'(c)$ could be quite flat and may be increasing as income increases. After a certain point, $z'(c)$ would then be diminishing again. This alternative formulation may in general be more realistic. Here we focus on the $z(c)$ concave case for simplicity of exposition but in future applications an S-shaped $z(c)$ may prove more appropriate.

[20] While these differences are large, they are obviously not large enough for hours differences alone to produce the income differences.

[21] In other words we assume that the gross interest rate is 1.

The equivalent of what we called the tax before is the term $1 - \beta c_2'(w_2)$. We are interested in the conditions under which $c_2'(w_2)$ is decreasing in $w_2$. From the second period decision-maker's first order condition,

$$c_2'(w_2) = \frac{\beta\delta U''(c_3)}{U''(c_2) + \beta\delta U''(c_3)}.$$

Differentiation again yields

$$\begin{aligned}
& c_2''(w_2) \\
= & \frac{1}{[U''(c_2) + \beta\delta U''(c_3)]^2}\{\beta\delta U'''(c_3)(1 - c_2'(w_2))[U''(c_2) + \beta\delta U''(c_3)] \\
& -\beta\delta U''(c_3)[U'''(c_2)c_2'(w_2) + \beta\delta U'''(c_3)(1 - c_2'(w_2))]\} \\
= & \frac{\beta\delta U'''(c_3)(1 - c_2'(w_2))U''(c_2) - \beta\delta U''(c_3)U'''(c_2)c_2'(w_2)}{[U''(c_2) + \beta\delta U''(c_3)]^2}
\end{aligned}$$

In other words, $c_2''(w_2) < 0$, iff $\frac{U'''(c_3)}{[U''(c_3)]^2} < \frac{\beta\delta U'''(c_2)}{[U''(c_2)]^2}$. The hyperbolic model equivalent of $U$ utility being less concave than $V$ utility is a property regarding how $U'''(c)$ changes relative to the square of $U''(c)$, which is not a very intuitive relationship.

# 4 Implications of the model

## 4.1 Attributions of Impatience

A simple observation trivially follows from our key assumption. Recall that in subsection 2.4 we had defined $\widehat{\delta}$ to be the discount factor that an observer would (mistakenly) attribute to our decision-maker, if he assumes a model with no dropped utility. In the case where there is no uncertainty,

$$\widehat{\delta} = \delta[1 - \frac{dz^2}{dc^2}].$$

Since $z^2$ is assumed to be concave as a function of $c^2$, this tells us that those who are richer, in the sense of consuming more in the second period, will appear to be more patient to the observer despite the fact that everyone has the same $\delta$. As will be shown later, second period consumption is monotonic in first period total income, and hence this could also be stated in terms of first period income (and also in terms of second period income).[22]

This framework, therefore, suggests an intuitive re-interpretation of the common observation that the poor seem to be more myopic than the non-poor that goes back at least to Irving Fisher (1932). This is a direct consequence of declining temptations. As we see in the section below, there is a secondary follow-on consequence. When the poor give into their temptations, it simply has larger

[22]We put it in terms of second period consumption because, as will emerge, first period consumption is not necessarily monotonic in first period wealth.

consequences: In this framework, z(c) concave means, that it takes a smaller fraction of resources to satisfy one's temptations at high level of consumption than it does to satisfy one's temptations at a low level of consumption.[23] All this goes to say, suppose we examine the behaviors of two people with identical discount rates, one of whom was born well off and the other poor. The poorer one will appear more myopic if born poor simply because the same failures (giving in to temptations) have greater consequences when poor. This last statement, of course, is a combination of the proposition above and a (yet to be proved) result of how wealth distributions evolve in the presence of declining temptations. We make this statement precise in Section 4.4 below.

## 4.2 Chaining

For simplicity, we have focused on the two-period case. The two period case masks a magnification that takes place over time. This magnification, which we refer to as chaining, is especially important for quantifying the importance of temptation. Here we add a third period to illustrate this chaining effect. We begin with a 3 period version of the T-period model introduced above and simplify it by assuming that the person has no prospect for production ($F(k^1, \theta) = 0$) and no second period income. His only source of second period income is to save at the given gross interest rate 1.

We solve through backward induction.The third period self simply maximizes $U(x_3)+V(z_3)$ subject to $x_3+z_3 = w_3 = w_2-c_2$. This defines $x_3(w_3) = x_3(w_2-c_2)$. Second period self now maximizes $U(x_2(c_2))+V(z_2(c_2))+\delta U(x_3(w_2-c_2))$, which gives us $c_2(w_2)$. Given the solution to this problem, the first period decision maker maximizes

$$U(x_1(c_1)) + V(z_1(c_1)) + \delta U(x_2(c_2(w_1 - c_1)) + \delta^2 U(x_3(w_2 - c_2(w_1 - c_1)))$$

In deciding how much to consumer, period 1 self trades off utility today from consumption $(U(x_1(c_1)) + V(z_1(c_1))$ and the future utility that savings would provide. Define $W_u(\cdot)$ to be this future utility:

$$W_u(w_1 - c_1) = U(x_2(c_2(w_1 - c_1)) + \delta U(x_3(w_2 - c_2(w_1 - c_1)))$$

The marginal benefit of savings is given by differentiation:

$$\begin{aligned} & W_u'(w_1 - c_1) \\ = \; & U'(x_2)x_2'(c_2)c_2'(w_1 - c_1) \\ & +\delta U'(x_3)x_3'(c_3)[1 - c_2'(w_1 - c_1)] \\ = \; & U'(x_3)x_3'(c_3) - \\ & [\delta U'(x_3)x_3'(c_3) - U'(x_2)x_2'(c_2)]c_2'(w_1 - c_1) \end{aligned}$$

where $c_3 = w_2 - c_2(w_1 - c_1)$.

[23] Note that although we do not model it here, one could include a self-control technology here. In that language, we would say that the poor require *greater* self-control since they would need to resist giving into the same temptations more than the rich.

But from the first order condition for the period 2 self's maximization,

$$U'(x_2) = \delta U'(x_3)x_3'(c_3)$$

which means the above expression can be rewritten as

$$\begin{aligned} W_u'(w_1 - c_1) &= \delta U'(x_3)x_3'(c_3) \\ &\quad -U'(x_2)(1 - x_2'(c_2))c_2'(w_1 - c_1) \end{aligned}$$

Compare this to the case where there is only one future period in the slightly artificial sense that the period 1 self controls $c_2$ (as well as $c_1$) but not $x_2$ or $x_3$. In this case, the reward to saving is given by the function

$$W_u(w_1 - c_1) = \max_{c_2} U(x_2(c_2) + \delta U(x_3(w_2 - c_2))$$

From the envelope theorem we have the reward for an extra dollar of saving in this case is

$$W_u'(w_1 - c_1) = \delta U'(x_3)x_3'(c_3).$$

Comparing this expression with the previous one makes clear that the reward for saving is less in the case where the period 1 self faces two independent future decision-makers than in the case where he faces one.

This is the sense in which there is "chaining" of temptations: the period 1 self is more tempted in the three period case because he knows that the period 2 self is also tempted, and will "waste" some of the resources that reach him before they get to the period 3 self.

## 4.3 Consumption smoothing

One of the most robust predictions of the standard model of savings is that an increase in future earnings ($y^2$) that leaves the return on investment unaffected will reduce today's savings and increase today's consumption ($c^1$). This is the direct result of the desire for consumption smoothing, induced by diminishing marginal utility. An increase in future income generates a desire to spend more both today and tomorrow. In our model, however, this need not be the case because there is a natural countervailing force. Notice that as $y^2$ rises, $\frac{dx^2}{dc^2}$ goes up and as a result, the right-hand-side of the Modified Euler equation could potentially even go up: The increased spending by the future self on $x^2$ increases today's self's desire to transfer income to the future and may even outweigh the effect of diminishing marginal utility.

**Proposition 3** *Assume that second period income, $y^2$, is deterministic. Under NDTC, consumption today is increasing in future income: $\frac{dc^1}{dy^2} > 0$. Under DTC this need not be the case i.e. we might observe $\frac{dc^1}{dy^2} < 0$ over some range of $y^2$. Moreover we will only observe this pattern for people for whom $y^1$ and $y^2$ are sufficiently small.*

***Proof.*** *Take the Modified Euler equation for this case*

$$\delta E_\theta\{\frac{df(w^1,\theta)}{dw^1}\frac{dU(x^2(c^2(\theta)))}{dx^2}\frac{dx^2(c^2(\theta))}{dc^2}\} = U'(x^1)$$

*where* $c^2(\theta) = f(y^1 - x^1 - z^1, \theta) + y^2$. *In the NDTC, for any value of* $\theta$ *an increase in* $y^2$ *keeping* $c^1 = x^1 + z^1$ *fixed, increases* $x^2 = c^2(\theta)$ *and therefore depresses* $U'(x^2(c^2(\theta)))$ *for every realization of* $\theta$. *Moreover it either depresses* $\frac{dx^2(c^2(\theta))}{dc^2}$ *or leaves it unchanged, for every realization of* $\theta$. *Since* $\frac{df(w^1,\theta)}{dw^1}$ *is unchanged for each realization of* $\theta$*, the left hand side is now less than the right hand side. Therefore* $x^1$ *has to go up to restore equality, and since* $U'(x^1) = V'(z^1)$, $z^1$ *must follow suit. Therefore a higher* $y^2$ *must be associated with a higher* $c^1$.

*In the DTC, i.e where* $\frac{dx^2(c^2(\theta))}{dc^2}$ *is increasing, the basic logic is very similar except that it is no longer obvious that the right hand side goes down. To see this, take this case where* $U(x) = \log x$. *In that case, the product*

$$\begin{aligned} & \frac{dU(x^2(c^2(\theta)))}{dx^2}\frac{dx^2(c^2(\theta))}{dc^2} \\ = \quad & \frac{1}{x^2(c^2(\theta))}\frac{dx^2(c^2(\theta))}{dc^2}. \end{aligned}$$

*Whether* $x^1$ *goes up or goes down turns on whether* $\frac{1}{x^2(c^2)}\frac{dx^2(c^2)}{dc^2}$ *is increasing or decreasing as a function of* $c^2$. *A sufficient condition for this is that* $x^2(c^2)$ *is log-convex for* $c^2 \leq \max_\theta f(y^1, \theta) + \overline{y}^2$, *where* $\overline{y}^2$ *is the ceiling of the relevant range of* $y^2$. *It is easy to check that there are log-convex functions which are non-negative valued and satisfy* $x^2(c^2) \leq c^2$ *on any given finite range of* $c^2$. *Therefore from proposition 1 we can find a* $V(z)$ *function which makes* $x^2(c^2)$ *is log-convex. In such cases* $c^1$ *wil be decreasing in* $y^2$.

*Finally it is clear from the argument above that in order for* $c^1$ *to be decreasing in* $y^2$, $x^2(c^2)$ *has to be sufficiently convex to outweigh the natural concavity of the utility function. However since* $x^2(c^2)$ *is bounded above by* $c^2$, *there is a limit to how convex* $x^2(c^2)$ *can be on a unbounded domain—for large enough values of* $c^2$, $x^2(c^2)$ *must be approximately linear. Therefore* $c^1$ *can only be decreasing in* $y^2$ *for sufficiently low values of* $y^1$ *and* $y^2$ *(since* $c^2$ *is increasing in* $y^1$ *and* $y^2$*).* ■

This is a striking conclusion. It tells us that those who are sufficiently poor might actually react to the prospect of future income growth by beginning to save more. Conversely savings may actually be lower in exactly those times when cash will be needed the most in the future: faced with falling future incomes, people may boost consumption. This offers a possible interpretation of the idea that aspirations matter.

This result can be understood from a different angle, one that offers an intuition that helps us understand other results below. Consider an individual with a time-consistent utility function $u(c^1) + \delta u(c^2)$. Suppose however, that instead of a investment technology that earns $f(w^1)$, he has only access to a technology that pays off $f(w^1)x'(f(w^1)+y^2)$. Consider now the impact of an increase in $y^2$. There is the usual consumption smoothing motive that encourages an increase in $c^1$. But here, however, there's an additional motive: the investment technology becomes more attractive: If the latter motive is strong enough, we might see the opposite of consumption smoothing.

## 4.4 Poverty Traps

This link between future income and savings has implications for first period income as well. First period income determines how much can be left for future selves. But since the savings invested for future consumption at time 1 ($w^1$) and income at time 2 ($y^2$) have similar effects, it is clear that a rise in $y^1$ could, in principle, have the same effect as an increase in $y^2$. If individuals start out with more income/wealth (which in this model are the same thing), they will be able to leave more to future selves. But this potentially creates a feedback effect: more wealth for time 2 means that $x'$ is higher in that period, which creates an even greater desire to leave wealth to time 2. This feedback effect can be the source of a poverty trap:

**Proposition 4** *Assume that there are no shocks i.e. that both $\theta_1$ and $\theta_2$ are constants. Then as long as we are in NTDC $c^2$ will be a continuous function of $y^1$. On the other hand in DTC, there may exist a $\overline{y}^1$, $0 < \overline{y}^1 < \infty$ such that $c^2$ jumps discontinuously upward at $\overline{y}^1$.*

Before we come to the main result, it is useful to observe that $c^2$ is always monotonic increasing with respect to $y^1$ and therefore the fact that $c^2$ jumps upwards if it jumps is automatic.

**Lemma 5** *$c^2$ is monotonically increasing as a function of $y^1$.*

**Proof.** (of Lemma 5) To see this, suppose to the contrary, there exists $y_0^1$ and $y_1^1$ such that $c^2(y_0^1) > c^2(y_1^1)$ but $y_0^1 < y_1^1$. Let the values $c^1$ corresponding to this strategy be $c^1(y_0^1)$ and $c^1(y_1^1)$. Clearly $c^1(y_0^1) < c^1(y_1^1)$. Now consider an alternative consumption strategy for the person at $y_0^1$ where he consumes $c^2(y_1^1)$ in the second period and sets $\widetilde{c}_0^1 = c^1(y_0^1) + \frac{c^2(y_0^1)-c^2(y_1^1)}{R}$. This must be dominated by what he actually chooses which implies that

$$\begin{aligned} & W(c^1(y_0^1)) - W(\widetilde{c}_0^1) \\ \geq \; & \delta U(x^2(c^2(y_1^1))) - \delta U(x^2(c^2(y_0^1))) \end{aligned}$$

On the other hand the person at $y_1^1$ clearly prefers the pair $\{c^1(y_1^1), c^2(y_1^1)\}$ to the alternative of consuming $\widetilde{c}_1^1 = c^1(y_1^1) + \frac{c^2(y_0^1)-c^2(y_1^1)}{R}$ in the first period and $c^2(y_1^1)$ in the second. Therefore

$$\begin{aligned} & W(\widetilde{c}_1^1) - W(c^1(y_1^1)) \\ \leq \; & \delta U(x^2(c^2(y_1^1))) - \delta U(x^2(c^2(y_0^1))) \\ \leq \; & W(c^1(y_0^1)) - W(\widetilde{c}_0^1). \end{aligned}$$

However since $c^1(y_0^1)) - \widetilde{c}_0^1 = \widetilde{c}_1^1) - c^1(y_1^1)$, this contradicts the strict concavity of $W$. ■

**Proof.** (of Proposition 4) Consider the maximization problem:

$$U(x^1) + V(z^1) + \delta U(x^2(f(y^1 - x^1 - z^1) + y^2))$$

subject to

$$x^1 \geq 0, z^1 \geq 0.$$

As long as we are in NDTC, so that $x^2(c^2)$ is weakly concave, the strict concavity of $U(\cdot)$ and the weak concavity of $f(\cdot)$ (and the fact that these are all strictly increasing functions) guarantees that $U(x^2(f(y^1 - x^1 - z^1) + y^2))$ is a strictly convex (and decreasing) function of $x^1$ and $z^1$. $U(x^1)$ and $V(z^1)$ are also strictly concave. These conditions together guarantee that we have a strictly convex maximization problem, which tells us that the maximizers, $x^1$ and $z^1$, are always unique and vary continuously as a function of the parameters of the problem, $y^1$ and $y^2$. Hence the result.

In DTC, on the other hand, this may not be true. To simplify the construction assume that $f(y^1 - x^1 - z^1) = R(y^1 - x^1 - z^1)$,with $\delta R > 1$. Also set $y^2 = 0$.

The way we will analyze this problem is by looking at how $c^2 - c^1$ behaves as a function of $y^1$. Clearly if $c^2$ is a continuous function of $y^1$, so is $c^1$ and $c^2 - c^1$.

Choose an $x^2(c^2)$ function which is convex (corresponding to the fact that we are in DTC) such that there exists a $c^*$, $\infty > c^* > 0$, with $\delta R \frac{dx^2(c^*)}{dc^2} = 1$. Because $\delta R \frac{dx^2(c^*)}{dc^2} = 1$, if $c^2(y^1) = c^*$, then $c^1(y^1) = c^2(y^1) = c^*$. However for this to be true, $x^1 = x^2(c^*)$, must satisfy the second order condition

$$U''(x^1) + \delta R^2 U''(x^2(c^*))[\frac{dx^2(c^*)}{dc^2}]^2 + \delta R U'(x^1) \frac{d^2x^2(c^*)}{d(c^2)^2} \leq 0$$

Since $x^1 = x^2(c^*)$ and $\delta R \frac{dx^2(c^*)}{dc^2} = 1$, this expression can be rewritten as

$$\begin{aligned} 0 \quad \geq \quad & \delta \frac{U'(x^2(c^*))}{x^2(c^*)} [\{x^2(c^*) \frac{U''(x^2(c^*))}{U'(x^2(c^*))}\}(\delta + 1) \\ & + \delta \frac{x^2(c^*)}{c^*} \{c^* \frac{\frac{d^2x^2(c^*)}{d(c^2)^2}}{\frac{dx^2(c^*)}{dc^2}}\}] \end{aligned}$$

$-x^2(c^*) \frac{U''(x^2(c^*))}{U'(x^2(c^*))}$ is the coefficient of relative risk aversion of the $U$ function and measures its degree of concavity. $c^* \frac{\frac{d^2x^2(c^*)}{d(c^2)^2}}{\frac{dx^2(c^*)}{dc^2}}$ is a similar measure of convexity for $x^2(c^*)$. This condition therefore requires that the $x$ function is not so convex as to overwhelm the concavity of $U$.

It is also clear that we can choose the $x^2(c^2)$ function such that it satisfies the conditions of proposition 1, but violates the second order condition above at $c^2 = c^*$. Which means that the maximization problem has a local minimum at $c^*$: $c^*$ cannot be the value of $c^2(y^1)$ for any $y^1$.

Therefore since $c^2(y^1)$ is an increasing function, either $c^2$ is discontinuous or $c^2(y^1) \to \bar{c} < c^*$ as $y^1 \to \infty$. But the latter case is impossible as long as we assume that

$$\frac{dx^2(c^2)}{dc^2} > \varepsilon > 0 \text{ for } c^2 \leq \bar{c}$$

and

$$U'(x) \to 0 \text{ as } x \to \infty.$$

This follows from the fact that if $c^2$ remains bounded above by $\bar{c}$, $x^1 \to \infty$ as $y^1 \to \infty$, and therefore the right hand side of the modified Euler equation goes to 0 (because of our assumption that $U'(x) \to 0$ as $x \to \infty$) while the left hand side remains bounded away from zero.

It follows that under these conditions, $c^2(y^1)$ must have a discontinuity and "jump over" $c^*$.

This result reinforces the discussion earlier about the myopia of the poor. Notice that here two individuals with *identical* discount rates but with different initial wealth levels can end up with very different levels of apparent patience: the initially poor agent will appear to be impatient and the initially rich one will appear to be patient. ■

## 4.5 Precautionary savings

With standard preferences, it is well-known that an increase in income uncertainty in the second period (i.e a mean preserving spread in the distribution of $y^2$) will increase savings in a safe asset as long the single-period indirect utility function exhibits prudence. In our environment the single-period indirect utility function would be given by

$$W(c) = \max_x U(x) + V(c - x).$$

If we were to assume that there is a safe techonology for transfering wealth across time ($f^2(w^1, \theta) = Rw^1$, where $R$ is a constant), the condition for there to be precautionary savings would be $W'''(c) > 0$. A sufficient condition for that is that both $U$ and $V$ have non-negative third derivatives (and at least one of them is strictly positive).

With our kind of preferences this condition is no longer sufficient. To see this recall the modified Euler equation for this case:

$$\delta R E_{\theta'}\{\frac{dU(x^2(c^2(\theta')))}{dx^2}\frac{dx^2(c^2(\theta'))}{dc^2}\} = U'(x^1).$$

Following the logic of precautionary savings in the standard model, it is clear that in this case, a mean preserving spread in $c^2$ will lead to lower $x^1$ if the function

$$H(c) = \frac{dU(x(c))}{dx}\frac{dx((c))}{dc}$$

is convex as a function of $c$. Taking derivatives twice (assuming differentiability) we can write

$$H''(c) = \frac{d^3U(x)}{dx^3}[\frac{dx((c))}{dc}]^3 + 3\frac{d^2U(x(c))}{dx^2}\frac{d^2x((c))}{dc^2}[\frac{dx((c))}{dc}]^2 + \frac{dU(x(c))}{dx}\frac{d^3x((c))}{dc^3}.$$

Several things become clear from this expression. First we do have the standard precautionary savings effect coming in as long as $\frac{dU(x(c))}{dx}$ is convex as a function of $c$ : this is captured by the first term in the above expression, which is positive if $\frac{dU(x(c))}{dx}$ is convex. However, there are two additional, potentially countervailing effects. One comes from the fact that $\frac{dx(c)}{dc}$ may not be convex as a function of $c$. This is what the last term says. The other comes from the correlation between $\frac{dU(x(c))}{dx}$ and $\frac{dx(c)}{dc}$ which may not be positive: This is the middle term above.

Indeed under DTC $\frac{dU(x(c))}{dx}$ and $\frac{dx(c)}{dc}$ move in opposite directions when $c$ goes up and hence the middle term is always negative. This has a very intuitive explanation: The whole point of precautionary savings is to raise $x$ consumption levels in the state of the world when $c^2$ is particularly low. But in the DTC $\frac{dx(c)}{dc}$is particularly low when $c$ is low, and as result, saving more does not help very much in terms of raising $x$ consumption in when $c$ is low. Therefore, the DTC partially defeats the purpose of saving more to protect against low $c^2$ states.[24] Notice that the discussion so far has presumed differentiabilty of $x(c)$.In particular we will focus on the case where there is no poverty trap. Hence the failure of NDTC consumers to hold precautionary savings is independent of the poverty trap proposition above.

To summarize, we have the following Proposition:

**Proposition 6** *Assume that the distributrion of $y^2$ is described by a family $G(y, \gamma)$ which has support on $[\underline{y},\overline{y}]$, with increases in the scalar $\gamma$ representing mean preserving spreads in the distribution of $y$. Define $x^1(\gamma)$ to be the optimal choice of $x^1$ for each value of $\gamma$, and assume that in an open neighborhood of $\gamma^*$, $N_{\gamma^*}, x^1$ is differentiable as a function of $\gamma$. Define $c^2(y^2, \gamma) = R(y^1 - x^1(\gamma)) + y^2$ and assume that for $\gamma \in N_{\gamma^*}$, the values of $c^2(y^2, \gamma)$ fall into the non-empty interval $[\underline{c}, \overline{c}]$. Then under NDTC, as long as $U'''(x(c)) \geq 0$ and $\frac{d^3x((c))}{dc^3} \geq 0$ for $c \in [\underline{c}, \overline{c}]$, for any $\gamma \in N_{\gamma^*} > \gamma^*$, $x^1(\gamma) < x^1(\gamma^*)$, i.e a mean preserving spread in the distribution of $y^2$ reduces $x^1$. On the other hand under DTC, even if $\frac{d^3x((c))}{dc^3} \geq 0$, $U'''(x(c)) \geq 0$, and $V'''(z(c)) \geq 0$ for $c \in [\underline{c}, \overline{c}]$,it is possible that a mean preserving spread in the distribution of $y^2$ raises $x^1$.*

**Proof.** The proof that under NDTC, first period consumption goes down when $y^2$ becomes more uncertain, is exactly the same as the proof of a precautionary demand for savings as long as the function $H(c) = \frac{dU(x(c))}{dx}\frac{dx((c))}{dc}$, defined above, is convex. From the expression derived above, this is always true when $\frac{d^3x((c))}{dc^3} \geq 0$ for $c \in [\underline{c}, \overline{c}]$.

[24] On the other hand, the presence of this term also means that in the NDTC, we can get a precautionary savings effect even when $\frac{d^3U(x)}{dx^3} = 0$, since under NDTC $\frac{d^2U(x(c))}{dx^2}$ and $\frac{d^2x((c))}{dc^2}$ are both negative and hence the product is positive.

To prove that this condition does not guarantee the same result under DTC, consider the following example.

Let $U(x) = \ln x$ and $x(c)$ be described

$$\begin{aligned}
x(c) &= \alpha c, c \leq \underline{c}, 0 < \alpha < 1 \\
x(c) &= Ae^{\beta c}, \overline{c} > c > \underline{c} \\
x(c) &= \widetilde{\alpha} c - \gamma, c \geq \overline{c}, 0 < \widetilde{\alpha} < 1, \gamma > 0
\end{aligned}$$

To ensure $x(c)$ is differentiable, continuous and increasing everywhere, assume that

$$\begin{aligned}
\alpha \underline{c} &= Ae^{\beta \underline{c}} \\
\alpha &= \beta Ae^{\beta \underline{c}} \\
\widetilde{\alpha}\overline{c} &= Ae^{\beta \overline{c}} - \gamma \\
\widetilde{\alpha} &= \beta Ae^{\beta \overline{c}}
\end{aligned}$$

which together imply that $\overline{c}$ has to be equal to $\frac{1}{\beta}$ and that $\alpha < \widetilde{\alpha}$. Under these assumptions

$$\frac{dx(c)}{dc} = \beta x \text{ for } \underline{c} < c^2 < \overline{c}.$$

Therefore $\frac{dx^2(c^2)}{dc^2}$ is increasing as long as $c^2$ is between $\underline{c} < c^2 < \overline{c}$, and constant otherwise. We assume that at $\gamma^*$ the distribution of $y^2$ is such that $\underline{c} < c^2 < \overline{c}$. for all realizations of $\theta'$. Then this will continue to be true for a small perturbation in the distribution of $y^2$ and we can assume that $\frac{dx^2(c^2)}{dc^2} = \beta x^2$ (and therefore DTC) everywhere in the relevant range. Moreover

$$\frac{dx^3(c)}{dc^3} = \beta \frac{dx^2(c)}{dc^2} \geq 0$$

as required by the condition of the Proposition.

Substituting this in the modified Euler equation gives us

$$\begin{aligned}
\delta R E_{\theta'} \{ \frac{1}{x^2(c(\theta'))} \beta x^2(c(\theta')) \} &= U'(x^1) \\
&\quad \mathit{or} \\
\beta \delta R &= U'(x^1)
\end{aligned}$$

In other words, the right hand side is now a constant: Shifts in the distributionof $y^2$ (mean preserving or otherwise) have no effect on the decision to save as long as $c^2$ remains in the relevant range.

To complete the proof we need to show that $U'$ and $V'$ are convex. Now, $U(x) = \ln x$, so $U''' > 0$. $V(z)$ has to be defined to generate the chosen $x(c)$ function:

$$V(z) = \int_0^z U'(g(y)) dy$$

where $x = g(z)$ is the relationship between the optimizing values of $x$ and $z$ for the same value of $c$. Therefore

$$\begin{aligned}
V'(z) &= U'(g(z)) \\
V''(z) &= U''(g(z))g'(z) \\
V'''(z) &= U'''(g(z))[g'(z)]^2 + U''(g(z))g''(z)
\end{aligned}$$

Now since $\frac{dx}{dc} = \beta x$ and $\frac{dz}{dc} = 1 - \beta x$,

$$\begin{aligned}
g'(z) &= \frac{dx}{dz} = \frac{\beta x}{1 - \beta x} \\
& \quad \textit{and} \\
g''(z) &= \frac{dx}{dz} = \frac{\beta}{(1 - \beta x)^2} \frac{\beta x}{1 - \beta x}
\end{aligned}$$

Therefore

$$\begin{aligned}
V'''(z^2) &= \frac{2}{(x^2)^3} (\frac{\beta x^2}{1 - \beta x^2})^2 \\
& \quad - \frac{1}{(x^2)^2} \frac{\beta}{(1 - \beta x^2)^2} \frac{\beta x^2}{1 - \beta x^2} \\
&= \frac{\beta^2}{x^2 (1 - \beta x^2)^2} [2 - \frac{1}{1 - \beta x^2}]
\end{aligned}$$

This is positive in the relevant range as long as

$$\frac{1}{1 - \beta x(\overline{c})} < 2.$$

Moreover for $c^2$ outside the range $[\underline{c}, \overline{c}]$ $z$ is linear in $x$, and therefore $V'''$ has the same sign as $U'''$. Therefore the condition for precautionary savings with conventional preferences (that both $U'''$ and $V'''$ are strictly positive) holds everywhere. Yet there is no precuationary savings. ■

A few points about this result are worth emphasizing. First, the condition that $x^1(\gamma)$ is differentiable is imposed for expositional purposes. Under NDTC, we know that $x^1(\gamma)$ is always continuous as a function of the underlying parameters and therefore it is always possible to establish a variant of this proposition for that case without any differentiability. Under DTC, we are only trying to find a counterexample, and allowing for the possibility that $x^1(\gamma)$ can be discontinuous only makes easier to contruct such an example. Second, the condition that $\frac{d^3 x((c))}{dc^3} \geq 0$ is necessary to get the NDTC result. It is possible to construct examples where $\frac{d^3 x((c))}{dc^3} < 0$, and there is no precuationary savings even under NDTC.

## 4.6 Implications for the Structure of Investments

Standard utility theory has some clear and useful implications about the nature of investment demand. Specifically it tells us that while the minimum scale of a project is a consideration (because of credit constraints), in addition to the rate of return (we assume there is no risk), the *maximum* scale of investment is not a consideration. You may not go all the way up to the maximum scale, but the fact that there is a maximum scale is irrelevant as long as the project has a high enough mean return. This is no longer necessarily true in our model of temptations: Individuals facing declining temptations will be unwilling to invest in high returns investments if the scale is too small. This captures the lay intuition that investments may be unimportant unless they significantly change one's circumstances.

Assume that the period one self has been offered access to a set of new investment technologies (in addition to what was already available to him). We are interested in whether he would take it up. These new investments are described by three features. Assume that each investment $\iota$ is described by its return $R(\iota)$, minimum size $s(\iota)$, and maximum size $S(\iota)$.[25] If he undertakes the investment $\iota$, he can choose an investment level $I$ between $s(\iota)$ and $S(\iota)$. Period 1 self will then have $I$ units less to spend but period 2 self will receive $R(\iota)I$ returns. In this highly abstract setup, we can examine the rank-ordering of investments. If individuals are willing to undertake $\iota$ what can we say about any other $\iota'$ that they would also be willing to undertake?

To analyze this rigorously we focus on a setting where both $y^1$ and $y^2$ are deterministic, borrowing is ruled out (allowing some borrowing would not change anything essential, but makes some of the arguments more tedious) and there is "base" investment technology, $f(w^1) = R_0 w^1, R_0 > 1$. The investor is now offered the option of investing in *one* additional technology with the understanding that he still has the option of investing as much as he wants in the base technology (subject, as before, to the constraint that he cannot borrow).

**Proposition 7** *In this setting under NDTC if the investor is willing to undertake an investment $\iota = \{R(\iota), s(\iota), S(\iota)\}$ then he will always be willing to undertake an investment $\iota' = \{R(\iota'), s(\iota'), S(\iota')\}$ as long as $R(\iota') \geq R(\iota)$ and $s(\iota') \leq s(\iota)$. In other words, minimum scale and returns summarize the investment. In contrast, if $S(\iota) > S(\iota')$, under DTC there exist situations where this is not true even if $R(\iota') > R(\iota)$ and $s(\iota') < s(\iota)$ .*

***Proof.*** *Define $W(c^1, c^2)$ to be the period 1 self's maximand, i.e.*

$$W^1(c^1, c^2) = W(c^1) + U(x^2(c^2)).$$

*In the NDTC, since $x^2$ is a concave function of $c^2$, $W^1(c^1, c^2)$ is a strictly concave function of the vector $(c^1, c^2)$.*

*Suppose period 1 self's optimal choice when offered the option $\iota$ is to invest an amount $I > 0$ in $\iota$ and to consume an amount $c^1$ in period 1. The amount he invests in the base technology is therefore $y^1 - I - c^1$. Clearly in the absence of credit $y^1 - I - c^1$ must be non-negative. $c^2$ in this case is given by*

$$c^2 = IR(\iota) + (y^1 - I - c^1)R_0 + y^2$$

[25] The model also has implications for the timing of investment which we do not investigate in this current version.

*If instead he had chosen not to invest in* $\iota$, *he would have consumed an amount* $\widetilde{c}^1$ *in period 1 and invested* $y^1 - \widetilde{c}^1$ *in the base technology. Therefore*

$$\widetilde{c}^2 = (y^1 - \widetilde{c}^1)R_0 + y^2.$$

*By the fact that the investor chose to invest*

$$W(c^1, c^2) \geq W(\widetilde{c}^1, \widetilde{c}^2)$$

*Now consider the vector* $(\lambda c^1 + (1-\lambda)\widetilde{c}^1, \lambda c^2 + (1-\lambda)\widetilde{c}^2), 1 > \lambda > 0:$ *Clearly, by the strict concavity of W,* $W(\lambda c^1 + (1-\lambda)\widetilde{c}^1, \lambda c^2 + (1-\lambda)\widetilde{c}^2) > W(\widetilde{c}^1, \widetilde{c}^2)$. *The question is whether* $(\lambda c^1 + (1-\lambda)\widetilde{c}^1, \lambda c^2 + (1-\lambda)\widetilde{c}^2)$ *is in the option set. Note however that we can write*

$$\begin{aligned}
& \lambda c^2 + (1-\lambda)\widetilde{c}^2 \\
= \; & \lambda(IR(\iota) + (y^1 - I - c^1)R_0 + y^2) \\
& + (1-\lambda)((y^1 - c^1)R_0 + y^2) \\
= \; & \lambda IR(\iota) + (y^1 - \lambda I - c^1)R_0 + y^2
\end{aligned}$$

*In other words as long as the investment level* $\lambda I$ *is in the option set (i.e,* $s(\iota) \leq \lambda I \leq S(\iota)$*)* $(\lambda c^1 + (1-\lambda)\widetilde{c}^1, \lambda c^2 + (1-\lambda)\widetilde{c}^2)$ *is feasible and dominates just investing in the base asset.* ■

*Now consider an alternative asset* $\iota'$ *with returns* $R(\iota') \geq R(\iota)$. *Clearly*

$$\begin{aligned}
& W(\widetilde{c}^1, \widetilde{c}^2) \\
< \; & W(\lambda c^1 + (1-\lambda)\widetilde{c}^1, \lambda c^2 + (1-\lambda)\widetilde{c}^2) \\
\leq \; & W(\lambda c^1 + (1-\lambda)\widetilde{c}^1, \lambda IR(\iota') + (y^1 - \lambda I - c^1)R_0 + y^2)
\end{aligned}$$

*which is what you would get by investing the same amount* $(\lambda I)$ *in asset* $\iota'$ *after choosing the same amount of first period consumption. Therefore as long as we can make sure that the amount invested in asset* $\iota'$ *is feasible, i.e set* $\lambda I$ *between* $s(\iota')$ *and* $S(\iota')$, *then there will be investment in* $\iota'$. *But since* $s(\iota') \leq s(\iota)$, *it will always be possible to set* $\lambda$ *so that* $s(\iota') \leq \lambda I \leq S(\iota')$. *Therefore there will always be some investment in* $\iota'$.

*Under DTC,* $W(c^1, c^2)$ *is not necessarily concave. To see where things might break down when* $W(c^1, c^2)$ *is not concave consider the following special preferences:*

$$\begin{aligned}
V(z) &= az, z \leq \overline{c}, \\
&= a\overline{c}, z > \overline{c} \\
& \quad \textit{and} \\
U(0) &= 0, 0 < U'(0) < a, U''(x) < 0
\end{aligned}$$

*In the second period, these preferences imply that*

$$\begin{aligned}
z^2(c^2) &= c^2, z \leq \overline{c} \\
&= \overline{c}, z > \overline{c}.
\end{aligned}$$

*Given these preferences anyone with $y^1$ and $y^2$ such that $R_0 y^1 + y^2 < \overline{c}$, will not save as long as the base technology is the only available technology, because he faces a $z'(c)$ of 1. Moreover, given that $R_0 > 1, y^1$ must be less than $\overline{c}$ and therefore period 1's self will consume all of $y^1$ in the form of the $z$ good. His two period utility is therefore $ay^1$.*

*Next, assume that he is willing to invest the entire amount amount $y^1$ in technology $\iota$. This requires that*

$$R(\iota)y^1 + y^2 > \overline{c}$$

*that*

$$\delta U(R(\iota)y^1 + y^2 - \overline{c}) > ay^1.$$

*and that*

$$\delta R(\iota)U'(R(\iota)y^1 + y^2 - \overline{c}) > a.$$

*Clearly we can find a $R(\iota)$ large enough for which these conditions hold.*

*Finally assume that there is an $\iota'$ such that $R(\iota') > R(\iota)$, $s(\iota') < s(i)$ and $S(\iota') < S(\iota)$. Now if*

$$R(\iota')S(\iota') + y^2 < \overline{c}$$

*there will obviously not be any investment in $\iota'$,even though it has a higher per dollar return and lower minimum scale. The logic of this construction makes clear that it can easily be extended to the case where both $V$ and $z^2(x^2)$ are differentiable functions.*

This result is important, we feel, for two reasons. First, some of the high return investments which have been brought up in the literature as instances of a puzzling unwillingness to invest are naturally capped. Fertilizer, for example, may earn very high rates of return (Duflo, Kremer and Robinson (2009)) and has no obvious minimum scale but the maximum scale at which it can be applied is capped by the amount of land you own. Similarly, Kremer, Lee and Robinson (2009) argue convinvicingly that stocking behavior on phone cards is an example of an unexploited high return investment, but once again these are investments that are limited in terms of maximum scale: The optimal stocking pattern would involve holding a few more cards, but not a lot more. Second, even when projects have no natural maximum scale, the presence of credit constraints makes them have one. Therefore, even if land is a constraint on how much the farmers studied by Duflo, Kremer and Robinson (2009) invest in fertilizer, credit might be: Farmers might prefer to consume everything they have, because given the credit constraints, any interesting projects are beyond their reach. For both of these reasons, empirical work on investment decisions should also pay attention to the maximum feasible scale.

## 4.7 The Role of Credit

Declining temptations also have implications for the benefits and costs of credit. In the traditional model, access to credit is clearly good: it increases the opportunity set. In models with self-control problems, credit can potentially hurt.

Specifically, today's self could be made worse off if tomorrow's self has access to credit. This general feature of self-control models has more specific implications if we focus on declining temptations.

To fully capture commitment benefits in this context, we need to introduce a third ("zero") period. In the existing two period model, commitment has benefits only if the first period can restrict the *type* of consumption (x goods rather than z goods) in the second period. But in this section and those that follow, we will be interested in what is effectively a much cruder commitment mechanism: the restriction of overall level of consumption. In two periods, this would be a meaningless concept since the second period self would consume all the wealth in any case. In a three period model, however, the zero period self can undertake actions that restrict period one self's ability to consume.

Since we are interested only in the investment and commitment demands of this period zero self, we assume this self has no consumption. Instead he or she merely maximizes $U(x^1) + \delta U(x^2)$.[26] The other two periods are as before.

In this subsection, we focus on the case where the zero period self has a single decision: whether or not to allow the period one self to have access to consumption credit. The period 1 self has no automatic access to credit markets but the period zero self may give him specific types of access. The point of credit is to allow period 1 self to move consumption from period 2 to period 1; there are investment opportunities or opportunities for lending. We are interested in the types of access that the period zero self would be willing to allow his future selves. Suppose that a loan $\lambda$ is defined by two characteristics: $r(\lambda)$, the interest rate and a *maximum* loan size $L(\lambda)$. If period one self is allowed a loan of type $\lambda$, he can choose to borrow some amount between 0 and $L(\lambda)$ and have the period two self repay $r(\lambda)$ times that amount.

We then ask question of the type: if the zero period self allows loan type $\lambda$, will he allow loan type $\lambda'$? The key insight is that the *shape* of temptation places structure on the demand for commitment. It is well-known that the presence of temptations that cause the period 1 self to over-consume would make the period zero self interested in placing a limit on how big the maximum loan size $L(\lambda)$ can be. What is more suprising is that in the presence of declining temptations the zero period self might have an additional interest in placing limits on how *small* it can be.

**Proposition 8** *Under NDTC, if the period zero self is willing to allow a loan $\lambda = \{r(\lambda), L(\lambda)\}$, he will always willing to allow loan $\lambda' = \{r(\lambda'), L(\lambda')\}$ as long as $r(\lambda) = r(\lambda')$ and $L(\lambda') \leq L(\lambda)$. Under DTC there will exist situations where he is willing to allow a loan $\lambda = \{r(\lambda), L(\lambda)\}$, but not a loan $\lambda' = \{r(\lambda'), L(\lambda')\}$ where $r(\lambda) = r(\lambda')$,but $L(\lambda') < L(\lambda)$.*

***Proof.*** *NDTC: In this case since $x^1(c^1)$ and $x^2(c^2)$ are both concave, Period 0's utility function*

$$\Pi_0(L) = U(x^1(y^1 + L)) + \delta U(x^2(y^2 - Lr(\lambda))))$$

[26] In what follows, what matters is that the period 0 self is effectively more patient than period 1 self, so technically the period 0 self could also maximize $U(x^1) + cV(z^1) + \delta[U(x^2) + cV(z^2)]$ for some $c > 0$ without changing our analysis in any signficant way.

*is concave as a function of $L$, the actual loan amount. Assume that period 0 self permits a loan product $\lambda = \{r(\lambda), L(\lambda)\}$, but not a loan $\lambda' = \{r(\lambda'), L(\lambda')\}$ where $r(\lambda) = r(\lambda') = r$,but $L(\lambda') = L' < L(\lambda) = L$.. This means that*

$$\begin{aligned}\Pi_0(\widetilde{L}) &= U(x^1(y^1 + \widetilde{L})) + \delta U(x^2(y^2 - r\widetilde{L})) \\ &\geq U(x^1(y^1)) + \delta U(x^2(y^2)) = \Pi_0(0)\end{aligned}$$

*but*

$$\begin{aligned}\Pi_0(\widetilde{L}') &= U(x^1(y^1 + \widetilde{L}')) + \delta U(x^2(y^2 - r\widetilde{L}')) \\ &< U(x^1(y^1)) + \delta U(x^2(y^2)) = \Pi_0(0)\end{aligned}$$

*where $\widetilde{L}, \widetilde{L}'$ are the loan amount actually chosen under $\lambda, \lambda'$. Now any loan amount that is less than $L'$ could have been chosen both under under $\lambda$ and $\lambda'$. Therefore if $\widetilde{L} \leq L'$, there is no reason why a decision maker who allows $\lambda$ would be unwilling to allow $\lambda'$ since $\widetilde{L}$ will be chosen under both. The interesting case is where $\widetilde{L} \geq L' \geq \widetilde{L}'$. In this case $\widetilde{L}'$ is between $\widetilde{L}$ and a loan size of 0. But then, by the concavity of $\Pi_0(L)$, $\Pi_0(\widetilde{L}')$ has to be no less than $\Pi_0(0)$, which directly contradicts what we said above. This contradiction proves that $\lambda'$ will be allowed if $\lambda$ is allowed.*

*DTC: To show that this is not necessarily true in DTC, consider the example from the prevous sub-section where*

$$\begin{aligned}V(z) &= az, z \leq \bar{c}, \\ &= a\bar{c}, z > \bar{c} \\ & \quad and \\ U(0) &= 0, 0 < U'(0) < a, U''(x) < 0\end{aligned}$$

*In the second period, these preferences imply that*

$$\begin{aligned}z^2(c^2) &= c^2, z \leq \bar{c} \\ &= \bar{c}, z > \bar{c}.\end{aligned}$$

*Also set $r(\lambda) = 1$. Therefore*

$$x^1 + z^1 + x^2 + z^2 = y^1 + y^2$$

*Assume that $y^1 = \alpha\bar{c}$ $(0 \leq \alpha \leq 1)$, $y^2 = \bar{c} + k$ and there is no borrowing allowed. Call this scheme of $(1, 0) = \lambda_0$. Under this scheme, the period 1 self spends his entire income on $z$ while the period 2 self spends his first $\bar{c}$ on $z$ but the remaining $k$ goes to $x$. Therefore $\Pi_0(\lambda_0) = \delta U(k)$*

*Now assume that there exists a loan $\lambda_1 = (1, L(\lambda_1))$, where $L(\lambda_1) < (1-\alpha)\bar{c} < k$. For simplicity call this $\gamma\bar{c}$, $\gamma < 1 - \alpha$. Given the assumed preferences, the period 1 self will want to borrow as much as he can and spend it all on $z$. The pseriod 2 self spends the $k - \gamma\bar{c}$ that remains after satisfying his appetite for $z$ on $x$. Hence $\Pi_0(\lambda_1) = \delta U(k - \gamma\bar{c}) < \delta U(k)$.Therefore, the period 0 self does* *not stand to benefit (and actually is hurt by) any loan with a maximal loan size until the threshold of* $(1-\alpha)\,\overline{c}$ *and will refuse all of those.*

*A necessary condition for any loan he would allow is therefore* $L(\lambda) > (1-\alpha)\,\overline{c}$*. Assume* $L(\lambda_2) = (1-\alpha)\,\overline{c} + \varepsilon$*. Now the period 1 self actually spends an amount* $\varepsilon$ *on* $x$ *because he can satiate his desire for* $z$*. Therefore* $\Pi_0(\lambda_2) = U(\varepsilon) + \delta U(k - (1-\alpha)\,\overline{c} - \varepsilon)$*.*

*Now* $U(\varepsilon) \geq \delta\left[U(k) - U(k-(1-\alpha)\,\overline{c}-\varepsilon)\right]$*, for* $\alpha$ *sufficiently close to 1 since* $U$ *is strictly concave. Therefore we have shown that* $\exists\lambda_2 = (r(\lambda_2), l(\lambda_2), L(\lambda_2))$ *and* $\lambda_1 = (r(\lambda_1), l(\lambda_1), L(\lambda_1))$ *with* $r(\lambda_1) = r(\lambda_2)$*,* $l(\lambda_1) = l(\lambda_2)$*,* $L(\lambda_1) < L(\lambda_2)$ *where period 0 accepts* $\lambda_2$ *but not* $\lambda_1$*.* ■

The intuition behind this result is simple. When temptations are constant, the only concern for the zero-period self is over-borrowing. Over-borrowing hurts the period zero self in two ways: (i) Exaggerating the difference in consumption between the period one and two selves and (ii) Engaging in borrowing at a higher rate than the period zero self would want.[27] Self zero must weigh these costs against the potential for borrowing to facilitate investments. As a result, zero can only want to limit the possibility for borrowing. When temptations are declining, however, there is an offsetting force. When the period one self borrows a small amount, all of it might go into $z$, but when he gets to borrow more, declining temptations kick in and he spends more of the loan proceeds on $x$.[28]

These results, we feel, are helpful in helping us parse credit contracts that we observe. Consider two different contracts: microfinance and credit cards. Micro-finance offers only larger loans while credit cards only offer small loans. In the model with constant temptations, and no lumpy investment, the period zero self may accept to get a credit card but refuse a mcro-finance loan, but never the other way around. Declining temptations can explain why we may observe the opposite: resistence to credit cards (or the equivalent) while support for micro-credit.[29][30]

[27] Zero discounts at $\delta$ while 1 discounts at $\delta(1-z'(c))$. Interest rates between those would induce inefficient borrowing.

[28] Given that the problem comes from 1 taking a small loan and spending it mainly on $z$, it might seem that putting a floor on loan size would achieve the same goal in this environment. This is however not true: In this case a floor only matters if it is binding; if the period 1 self wants to borrow less, then the period 0 self would also want him to borrow less and would want to lower the floor.

A minimum loan size would help if, on the other hand, there were a lumpy investment opportunity (or a consumer durable) that period 1 self could, in principle, carry out. However the project is so big that even with the biggest available loan, carrying out the investment would require him to consume less today. In that case, since he values consumption today relative to consumption tomorrow more than the period zero self, he may be reluctant to carry it out even when period zero would want him to. In this situation offering the option of borrowing a small amount may not be in period zero's interest because period 1 is already overconsuming. On the other hand, a larger loan is useful to 1 only if he complements it by saving more, which is what 0 wants. However the logic of this effect makes it clear that it would also arise in the constant temptation case.

[29] However lumpy investment opportunities is an alternative explanation and one that, in addition, would explain why micro-finance loans have a minimum size.

[30] We do not explore here a different commitment feature of debt: the forced repayment that

## 4.8 Money lenders and debt traps

So far we have focused on the demand side: We now turn to the supply side of credit and investment opportunities. Consider an individual who is borrowing from a monopolistic money lender. Suppose a new investment arises that earns a rate of return higher than the money lender's cost of capital. If the money lender could price discriminate and offer a rate specifically for this investment, would the investment get made? Simple Coasian logic suggests it should. Yet there is a long tradition of arguing that the money lener has an incentive to block this investment. More broadly, it argues that money lenders in particular have an incentive to block progress either by opposing the introduction of technological improvements directly or by refusing to fund them (Bhaduri 1973). Ths logic is that the technological improvement will raise the earnings of those who used to borrow from the money-lender which in turn hurts the lender's profits through reduced borrowing. In short, the money-lender prefers to have his clients caught in a debt trap.

Clearly, if the technological innovation were to change the time structure of the borrower's earnings (so that his earnings become more present biased and therefore his incentive to borrow goes down) the money lender has every reason to feel threatened. However, as pointed out by Srinivasan (1994), if the investment requires spending in the present to increase future earnings, there is no reason why borrowing would go down and therefore the money lender should want to promote the innovation.

In the rest of this sub-section we will see that while the intuition proposed in the previous paragraph continues to hold under NDTC, the possibility of declining temptations reintroduces the possibility that Bhaduri had emphasized: The money-lender may indeed want block progress to keep the borrower in his thrall.

Assume that there is a time-inconsistent agent, who, as in previous subsection, lives for 3 periods but consumes $x$ goods and $z$ goods only in the last two periods. However the period 0 self only values $x$ consumption. Finally to limit the number of free parameters assume both the borrower and the lender do not discount future utilities, i.e., $\delta = 1$

The individual earns $y$ in each of the last two periods and the period 1 self has the option of saving at a gross interest rate of 1 and borrowing additional resources from a money lender at an interest rate $R_1$ set by the money lender. Then the period 1 self will choose his borrowing $L_1$ to maximize

$$W(y + L_1) + U(x(y - R_1 L_1)).$$

Assume that this is maximized by choosing $L_1(R_1)$

Suppose the agent borrows from a profit maximizing monopoly lender who can only set the interest rate (i.e he cannot set the amount borrowed as well) and whose (gross) opportunity cost of capital is $\rho > 1$ per period. The lender

it implies. We conjecture that on this dimension as well, contracts which appear as microfinance contracts (big lump sum, combined with small repayment installments) help solve temptation problems more so than credit card like contracts (small trickles of borrowing, combined with a big repayment.

therefore chooses $R_1$ to maximize $(L_1(R_1)R_1 - \rho L_1(R_1))$. Let $R_1^o$ be the interest rate that maximizes this expression. $\pi_1^0 = (L_1(R_1^0)R_1^0 - \rho L_1 R_1^0)$ is the maximized level of profit.

Now suppose an investment opportunity becomes available to the agent. It requires an investment of $I$ in period 0 and an action taken in period 0, which has a utility cost of $E$. The purpose introducing this cost is to make sure that the agent's lifetime earnings go strictly up if he undertakes the investment–in other words the interest rate the lender charges cannot be so high that the agent ends up with the same income after the investment than he had before, because in that case he would prefer not to put in the effort that the investment requires. The investment, if undertaken generates a revenue of $\phi I$ in period 2.

Now given that the agent has no resources in period 0, the investment requires a loan $L_0 = I$ in period 0. Suppose the money lender offers a loan of $I$ at interest rate $R_0$ in period 0 to be repaid when the investment pays off, i.e. in period 2.

However knowing that he will be richer in period 2, the agent may want to increase his consumption in period 1 by borrowing more from the money lender (he may also want to do the opposite, which is to prepay the loan he had taken out in period 0, but assume that the money-lender can block prepayment).

Consider the decision of how much to borrow in period 1. If the interest rate set by the money lender in period 1 is $R_1$, the agent will maximize

$$W(y + L_1) + U(x(y + (\phi - R_0)I - R_1 L_1))$$

which yields $L_1(R_0, R_1)$. Period 0 takes this decision as given and chooses between investing and not investing: He invests if

$$\begin{aligned} & U(y + L_1(R_0, R_1)) + U(x(y + (\phi - R_0)I - R_1 L_1(R_0, R_1))) - E \\ \geq \quad & U(y + L_1(R_1^o)) + U(x(y - R_1^o L_1(R_1^o))) \end{aligned}$$

The problem is interesting only if the investment is worth doing. As a feasibility condition we assume that

$$\begin{aligned} & U(y + L_1(\rho, R_1^o)) + U(x(y + (\phi - \rho^2)I - R_1^o L_1(\rho, R_1^o))) - E \\ \geq \quad & U(y + L_1(R_1^o)) + U(x(y - R_1^o L_1(R_1^o))) \end{aligned}$$

In other words, as long as the second period interest rate remains the same as what it is in the absence of investment, but the first period rate is set at the (two period) cost of capital, the project is worth doing. A necessary condition for this to be true is that $\phi > \rho^2$, since $E > 0$. Note that this feasibility condition is exactly the thought experiment at the beginning of the section: the new technology is profitable relative to the money lender's cost of capital.

Under this feasiblity condition, the lender always has the option of offering the agent an interest rate $\rho$ in period 0, and an interest rate $R_1^o$ in period 1 and getting him to do the project. He can also always block it by setting $R_0 = \infty$. The following result establishes that he will never want to block under NDTC but this is not true under DTC.

**Proposition 9** *In this setting under NDTC the moneylender will always be willing to lend the investor money to make the new investment possible. Under DTC there exist situations where this is not true.*

**Proof.** As noted above, under NDTC $U(x(y+(\phi-\rho^2)I-R_1^oL_1(\rho,R_1^o)))$ is an increasing and strictly concave function of $y+(\phi-\rho^2)I-R_1^oL_1(\rho,R_1^o)$, while $W(\cdot)$ is always increasing and concave. We use this to first show that when $I$ goes up from 0, keeping the interest rates unchanged, the borrower cannot strictly prefer to borrow less.

Suppose, to the contrary, the borrower borrows $L_1$ in period 1 when there is investment $I$ and $L_1'$ when there is no investment, and $L_1' > L_1$. By revealed preference it must be that

$$\begin{aligned} & W(y+L_1)+U(x(y+(\phi-\rho^2)I-R_1^oL_1)) \\ \geq\ & W(y+L_1')+U(x(y+(\phi-\rho^2)I-R_1^oL_1')) \end{aligned}$$

and

$$\begin{aligned} & W(y+L_1')+U(x(y-R_1^oL_1')) \\ \geq\ & W(y+L_1)+U(x(y-R_1^oL_1)) \end{aligned}$$

It follows from these two inequalities that

$$\begin{aligned} & U(x(y+(\phi-\rho^2)I-R_1^oL_1))-U(x(y+(\phi-\rho^2)I-R_1^oL_1')) \\ \geq\ & U(x(y-R_1^oL_1))-U(x(y-R_1^oL_1')). \end{aligned}$$

Since $\phi > \rho^2$, this contradicts the concavity of $U(x(\cdot))$ unless

$$\begin{aligned} & U(x(y+(\phi-\rho^2)I-R_1^oL_1))-U(x(y+(\phi-\rho^2)I-R_1^oL_1')) \\ =\ & U(x(y-R_1^oL_1))-U(x(y-R_1^oL_1')). \end{aligned}$$

which would imply that

$$\begin{aligned} & W(y+L_1)+(x(y+(\phi-\rho^2)I-R_1^oL_1)) \\ =\ & W(y+L_1')+U(x(y+(\phi-\rho^2)I-R_1^oL_1')) \end{aligned}$$

Therefore the agent cannot strictly prefer to borrow less.

Since the lender's revenue is

$$(R_0-\rho^2)I+(R_1-\rho)L_1$$

This means that the lender can make himself at least as well off when there is investment as when there is no investment, by setting $R_0=\rho^2, R_1=R_1^o$ (our feasibility condition gurantees that the borrower will invest when offered these rates), since he gets to lend more (or at least no less). He will therefore lend money to the borrower for investment.

To see what changes under DTC assume that

$$\begin{aligned} V(z) &= az, z \leq \overline{c}, a > 1 \\ &= a\overline{c}, z > \overline{c} \\ & \quad and \\ U(x) &= x \end{aligned}$$

Assume also that $\overline{c} > y$. Therefore, in the absence of the investment it is always optimal for the period 1 self to borrow the full amount of $y$ if he borrows anything at all, since he gains nothing from leaving any amount less than $\overline{c}$ for period 2 to consume. However period 1 does have the option of saving his period 1 income and spending it in period 2 – assume that the gross interest rate on savings is 1. By borrowing and consuming everything, he gets a utility of

$$a[y + \frac{y}{R_1}]$$

under the assumption that $y + \frac{y}{R_1} < \overline{c}$.[31] If he saves he gets

$$[2y - \overline{c}]$$

This puts an upper bound on $R_1$ implicitly given by

$$\frac{y}{R_1} \geq \frac{[2y - \overline{c}]}{a} - y$$

which only binds if $\frac{[2y-\overline{c}]}{a} - y > 0$. But since $\overline{c} > y$ and $a > 1$, $\frac{[2y-\overline{c}]}{a} - y$ must be negative, and therefore there is no constraint on how high $R_1$ can be. A monopolistic lender will set it to be infinite, and extract the borrower's entire second period income, i.e., $L_1 \simeq 0$. Therefore the lender will earn an amount $y$ in this case.

If the investment does take place the borrower faces a similar choice: He can either save his period 1 earnings and get utility $2y + (\phi - R_0)I - \overline{c}$ in period 2, in which case $L_1 = 0$, and the lender makes no money from $L_1$, though he may still make money from the zero period loan. We will return to this option in a few paragraphs.

The alternative for the lender is to make sure that the borrower borrows in period 1. By the way the preferences have been specified, only the marginal units of spending in period 2 go into $x$ consumption, which is all that period 1 cares about. Moreover the marginal utility of $x$ spending is constant. Therefore if the period 1 self is prepared to move one unit of spending from period 2 to period 1, he will want to do so for all the the other units of spending. In other words, if he borrows at all, he will borrow the entire amount $\frac{y+(\phi-R_0)I}{R_1}$.

Given that there is no $x$ consumption in period 2, the only value from the investment from the point of view of the period 0 self comes from period 1

[31] As we will show in the next few lines the solution to the monopolist's problem has $R_1 = \infty$ and as a result this condition is implied by $y < \overline{c}$.

consumption of $x$. Therefore it must be the case that there is some period 1 consumption of $x$ (otherwise the period 0 self will never agree to put in the effort). In other words, we must have

$$y + \frac{y + (\phi - R_0)I}{R_1} \geq \bar{c}.$$

Therefore the utility period 1 self gets from consuming his entire discounted income in period 1 has to be

$$[y + \frac{y + (\phi - R_0)I}{R_1} - \bar{c}] + a\bar{c}.$$

To make him willing to borrow, it must be that

$$\frac{y + (\phi - R_0)I}{R_1} + a\bar{c} \geq y + (\phi - R_0)I$$

Note that since $R_1 > 1$, increasing $R_0$ relaxes this constraint and makes it possible to raise $R_1$.

The lender's net earnings from $L_0$ and $L_1$ in this case are given by

$$\begin{aligned} & IR_0 - I\rho^2 + y + (\phi - R_0)I - \frac{\rho}{R_1}(y + (\phi - R_0)I) \\ = \quad & -I\rho^2 + y + \phi I - \frac{\rho}{R_1}(y + (\phi - R_0)I). \end{aligned}$$

For any fixed value of $R_1$, this expression is clearly increasing in $R_0$. Moreover as observed above, raising $R_0$ allows the lender to raise $R_1$. This gives the lender a double incentive to raise $R_0$. However in setting $R_0$ he has to consider period zero's incentive constraint. Period zero will only put in the necessary effort in this scenario if

$$[y + \frac{y + (\phi - R_0)I}{R_1} - \bar{c}] - E \geq 0.$$

which tells us that

$$\frac{y + (\phi - R_0)I}{R_1} \geq \bar{c} + E - y$$

Using this in the expression for the lender's profits gives us the following upper bound for the lender's profits

$$y + (\phi - \rho^2)I - \rho(\bar{c} + E - y).$$

Can this be less than $y$, which was the lender's profit in the absence of investment? Clearly this depends on how large $E$ can be. The constraint on $E$ comes from feasibility. Since $R_1^o = \infty$, the agent does not borrow in period 1, and the feasibility constraint is simply

$$2y + (\phi - \rho^2)I - \bar{c} - E \geq 0$$

Suppose this constraint is only just satisfied, so that

$$2y + (\phi - \rho^2)I = \overline{c} + E$$

Substituting this into the upper bound on the lender's profits, we get

$$\overline{c} + E - y - \rho(\overline{c} + E - y)$$

which is negative. Clearly there is a range of values of parameters for which the lender would never choose this option.

Finally consider the case where the lender gives up trying to lend in period 1 and simply tries to extract enough using $R_0$. In this case the lender's profit is simply

$$(R_0 - \rho^2)I.$$

The constraint on $R_0$ comes from period zero's incentive constraint, which is

$$\begin{aligned} 2y + (\phi - R_0)I - \overline{c} - E &\geq 0 \\ & \quad \textit{or} \\ 2y + \phi I - \overline{c} - E &\geq R_0 I \end{aligned}$$

Obviously in the case where the feasibility constraint is just satisfied, this reduces to the constraint $R_0 I \leq \rho^2 I$. The lender makes no money. Hence he would prefer to block the investment by setting $R_0$ very high. ■

A simple corollary follows immediately from this the proof of this proposition:

**Corollary 10** *Suppose the investment does not cost any money i.e $I = 0$. However the money-lender can somehow block the agent's access to the investment. Under NDTC he will never exercise this option. However under DTC there exist situations where he will choose to block access to the investment.*

## 4.9 Empirical Tests of the Model

The above propositions provide qualitative predictions. However, the model also provides a few specific quantitative tests. Here we outline how those tests might appear.

First, we can actually test whether temptation is in fact declining. To do this, suppose there are several goods $g_i$. Suppose that each good has an $x$ component and a $z$ component. Specifically suppose that one unit of good $i$ provides $x_i$ fraction of the non-temptation and $z_i$ fraction of the temptation good. Moreover, suppose that what constitutes $x$ and $z$ goods is common across individuals.[32] Now consider the following experiment. Suppose that we offer to a set of individuals a choice between 1 unit of good $i$ and $d_i$ units of good $i$ in one period. Assume, as is common in all discount rate experiments, that there

[32] A weaker assumption that would still work is to assume there is a common component with individuals varying in an iid way around this common component.

is non-fungibility across time and goods so that individuals view this offer as a genuine increase of either 1 unit today or $d_i$units tomorrow.[33] Define $\overline{d}_i$ to be the average $d_i$ that makes individuals indifferent to this tradeoff, i.e. half the individuals choose 1 unit today and half choose $\overline{d}_i$ units in one period. These experiments allow us to array goods according to how much temptation they provide. Goods that provide more temptation should show larger $\overline{d}_i$: one would need a large quantity in the future to induce one to give up one unit today. The key assumption of our model can be empirically tested by looking at the Engel curve for each good: declining temptation implies that the Engel curve should be steeper for goods with lower $\overline{d}_i$. Note that this is not a hard-wired assumption. The Engel curve captures how demand for a particular good varies with income, whereas $\overline{d}_i$ measures the discount rate associated with a particular good.

A second quantitative test of our model is based on our assertion that the apparent patience difference between poor and rich is due to the composition of consumption rather than genuine differences in patience. To test this, we would offer trade offs of 1 unit of money today versus $d_m$ units tomorrow and use this to back out an apparent discount rate for money $\delta_m$. We would then offer (again, under the assumption of fungibility), 1 unit of an $x$ good (identified as above) today versus $d_0$ units tomorrow. This allows us to back out an apparent discount rate for $x$ goods $\delta_x$. We then predict that $\frac{\delta_m}{\delta_x}$ is declining in income: the poor are much more impatient in money than in x goods and this gap closes as income increases.

Note that these procedures do not just provide a test of the model. They provide an important discipline to this approach. A key judgment needed to operationalize this model is a delineation of which goods are temptation goods. Such a delineation can be made particularly difficult since there can be large inter-personal differences in preferences. The above procedure provides a way to elicit an individual's *own* judgments about what constitutes a temptation good. Such judgments, we feel, will sometimes yield counterintuitive valuations. For example, when Ashraf, Karlan and Yin (2004) provide commitment savings accounts, they ask individuals what they are saving up to buy. Since one would only save up to buy $x$ goods, it is interesting to see that the second most common stated purpose for savings is festival and party expenditures. This contrasts to an outsider who might argue that the poor "waste" their money on these expenditures.[34] This example highlights the need for judicious use of experiments of the type above to determine what is a temptation good and what is a genuine oasis of pleasure.

[33] This assumption is common in all discount rate experiments that are undertaken for money, e.g. one dollar today or two dollars in one month. Such experiments are meaningless if money were fungible in time for example.

[34] Of course, to the extent that they generate a conspicuous consumption externality, there can still be an argument for social waste.

# 5 Conclusion

We feel that understanding the structure of temptations ("wasted expenditures") is essential to understanding the lives of the poor. Our framework highlights one important structural feature of temptations: whether their impact is declining in income. The results in this paper illustrate that declining temptations can help to explain a large range of phenomena, from poverty traps to credit and investment behavior. Though we do not focus on these issues in this paper, they may also be helpful in answering a broader set of questions. First, we mentioned briefly the idea that understanding the structure of temptation may help in classifying and designing commitment savings products. Should these products force savings or limit withdrawals? Should they put a ceiling on withdrawals (so that there is always something left against emergencies or limit withdrawals until a minimum amount is reached (so as to enable the accumulation of a large lump sum)? We understand little empirically or theoretically about these questions. Second, while our model focuses on expenditures, many of the important choices of the poor are not fully captured by a *pure* expenditure focus. For example, yhe labor leisure choice is an important component of overall income and wealth accumulation. Understanding how these choices integrate into a temptation framework seems particularly important. Finally, a deeper understanding of social behaviors, for example drinking and smoking, would be interesting but would require a greater investment in the technology of addiction as well as in the shape of temptation emphasized here. While the existing model makes some progress, much more interesting work remains to be done.

# 6 Appendix

Proof of Proposition 2

**Proof.** Since $U'(x^2(c^2)) = V'(z^2(c^2))$, and $x^2(c^2)+z^2(c^2) = c^2$, $\frac{dz^2}{dc^2} = \frac{U''(x^2(c^2))}{V''(z^2(c^2))+U''(x^2(c^2))}$ and $\frac{dx^2}{dc^2} = \frac{V''(z^2(c^2))}{V''(z^2(c^2))+U''(x^2(c^2))}$. Taking derivatives and substituting the values of $\frac{dx^2}{dc^2}$ and $\frac{dz^2}{dc^2}$ gives us

$$\frac{d^2z^2}{d(c^2)^2} = \frac{(V'')^2U''' - (U'')^2V'''}{(U''+V'')^3}$$

Now define $H$ such that $\widetilde{V}(x^2) = V(z^2(x^2)) = H(U(x^2))$. Taking derivatives and using the fact that $\frac{dz^2}{dx^2} = \frac{U''}{V''}$gives us that

$$H'(U) = \frac{V'U''}{U'V''}.$$

Taking derivatives again gives us that

$$H''(U) = \frac{1}{U'}[\frac{U'V''(V'U''' + V''U''\frac{dz^2}{dx^2}) - V'U''(U'V'''\frac{dz^2}{dx^2} + U''V'')}{(U'V'')^2}].$$

Since $V' = U'$, this reduces to

$$H''(U) = \frac{V'}{V''}[\frac{(V'')^2U''' - (U'')^2V'''}{(U'V'')^2}].$$

Since $(U''+V'')^3$ has the same sign as $V''$, the result follows. ■

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